



Development of Discrete Mathematics Module Based on Discovery Learning for Mathematical Understanding in Higher Education

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Abstract: This research aims to develop a discrete mathematics module based on discovery learning to improve students' mathematical understanding in higher education. This research used the Research and Development (R&D) method with a mixed approach and ADDIE model in the product development stage. The data collection techniques used questionnaires, tests and interview guides. Meanwhile, the data analysis technique used in this research combines qualitative and quantitative descriptive. The results include (1) Reviewing the study of discrete mathematics module development; it obtained valid results after going through several assessments and revisions, and the practicality of developing this discrete mathematics module was categorized as practical; it is not surprising that the module can increase students' mathematical understanding with a range of 37.97; (2) Judging from effectiveness; it is obtained from inferential statistics that significance is 0.00, which means there is an influence on the use of discovery learning-based discrete mathematics modules on students' mathematical understanding. These results have implications that the module has proved effective in improving students' mathematical understanding in higher education.

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Introduction

One mathematical skill or ability that is expected to be attained is understanding mathematical concepts. Demonstrating comprehension of the topics being studied, explaining the relationships between concepts, and applying concepts or algorithms in a flexible, accurate, efficient, and precise way when solving problems are a few examples of these abilities (Komarudin et al., 2021; Mawaddah & Maryanti, 2016). Students who grasp mathematical concepts are able to recognize and resolve a number of novel and difficult mathematical issues (Nasution, 2008). In mathematics learning, understanding a concept is a fundamental element in the learning process (Kosiret et al., 2021; Tan & Ang, 2016; Ainley & Ainley, 2011). Understanding mathematical concepts supports students' cognitive thinking processes (Virgana, 2019). Nowadays, mathematical understanding is needed to understand mathematical material, and mathematical reasoning can be trained and honed through learning mathematics (Kartono & Shora, 2020). Students who are able to control their thinking are better able to apply what they already know, plan and establish learning objectives, track their progress toward those objectives, assess the degree of understanding that has been created, and master the material covered in class (Bakar & Ismail, 2020; Schraw & Moshman, 1995; Nelson & Narens', 1990).

The discovery learning model is one type of instruction where students must figure out problems and mathematical concepts (Roza et al., 2018). Mathematical education has



long been concerned with discovery learning, both as a core activity and as an instructional principle (Loibl & Leuders, 2018). In addition, one of the learning models that encourages students to participate actively in designing successful lessons is thought to be discovery learning (Permatasari et al., 2019). Yurniwati and Hanum (2017) state that guided discovery learning is suitable for allowing students to discover and understand concepts in depth. Abrahamson and Kapur (2018) argue that discovery describes the dynamics of more complex thinking in the environment between the growth of conceptual knowledge and its transferability.

Students use the discovery learning model to investigate and resolve issues. The concept of problem-solving can hone students' abilities to understand concepts (Tricot & Sweller, 2013; Mchaney, 2012). Discovery learning is cooperative learning, where students work in small groups to improve their understanding of the material (Ott et al., 2018). Students will engage in discovery learning to the extent that it helps them develop their ability to analyze, interpret, and think critically (Siregar et al., 2020; Majid & Majid, 2018; Edwards, 2015). According to Dina et al. (2019), this approach works best when students are given a problem to solve and are expected to draw conclusions based on the fact-based outcomes of their learning.

Discovery learning considers learning a process for students to explore and experiment with the physical/material world by constructing hypotheses. This construction focuses on knowledge based on their previous experiences and beliefs (Korres, 2019). In the construction process, students are designed to work in structured small groups to advance their understanding of the material (Ott et al., 2018). The essence of competent teaching is the connection between educators and the educated people they guide to obtain teaching that is suitable for students to understand and interact with the world, and this approach is believed to improve student performance as measured through tests at the end of the learning process (Tanase, 2020). At this stage, it appears that the primary attraction of discovery is its close alignment with the concept of autonomy—the capacity for independent thought—which is frequently regarded as a crucial educational objective. It is true whether one is interested in learning and teaching science or any other related field (Bakker, 2018; Biesta, 2015; Bakhurst, 2011). One of the things required to achieve innovative development studies is to benefit students' mathematical understanding in the classroom.

Using modules is one effort to the learning of mathematics. The smallest unit of a teaching and learning program that is studied independently by students or taught by them is called a learning module (Winkel, 2009). The goal of this module development study is to make it easier for students to understand and assimilate the material they are learning (Setiyani et al., 2020). Students actively learn how to construct knowledge based on learning experiences through this module (Norton & Alibali, 2018; Von Glaserfeld, 1989). The learning activities are designed to help students become more proficient in attitudes, knowledge, and skills (Amir & Ihamuddin, 2021; Yuberti et al., 2019; Bao et al., 2018), including comprehending mathematical concepts. Numerous instructional strategies, such as discovery-based strategies, have been used to try to raise students' proficiency in mathematics (Siregar et al., 2019).

Numerous studies have demonstrated the efficacy of discovery learning-based modules in enhancing general science competencies. Statistical analyses have revealed noteworthy distinctions between the post-test scores of the control group, which utilizes commonly used modules in schools, and the treatment group, which utilizes modules based on learning discovery (Elok et al. 2017). In line with Wen and Meng (2021), the application of discovery learning effectively improved student achievement, which was carried out for

two experimental classes on fraction material. Module development was also carried out by Siregar et al. (2020), who stated that the D-Geometry module positively impacted students' mathematical reasoning, communication, and self-confidence. Furthermore, the research results of Jazim, Anwar, and Rahmawati (2017) state that the development of math modules encourages students to participate actively in class discussions and effectively develop their understanding of the subject matter.

Based on several of these studies, the state of the art in this research from previous research is for the effectiveness of studies on developing discrete mathematics modules based on discovery learning to improve mathematical abilities. Meanwhile, this research aims to develop a discrete mathematics module based on discovery learning in improving students' mathematical understanding in higher education

Research Method

This research used Research and Development (R&D) method with a mixed approach. The model design used in the module development technique was the ADDIE model with five development stages, namely Analysis (A), Design (D), Development (D), Implementation (I), and Evaluation (E). The data collection techniques used in this research function to answer the research objectives. The specifics are presented in Table 1.

Table 1. Research Data Collection Techniques

No	Research purposes	Data collection technique	Data source	Obtained data
1	Development of Discrete Mathematics Modules based on Discovery Learning	Questionnaire	Lecturer/Material, Language and Media Expert	Module feasibility/validity test data from material, language and media aspects
			Students	Data on student responses to module practical tests
			Lecturer	Study field teacher response data to the module practicality test
2	Effectiveness of Discrete Mathematics Modules Based on Discovery Learning for Students' Understanding of Concepts	Test	Students	Pre-test and post-test data of students after using the module

In this research, two tools (instrumental) were used in the research and development activities of this module, namely as follows:

a) Instruments in the form of tests

This assessment tool is used to gauge students' comprehension of mathematical ideas. In order to make sure the instruments being used are appropriate, the guidelines for using instruments in tests start with testing the prerequisites for the instruments. Meanwhile, Table 2 displays the prerequisite instrument tests.

Table 2. Validity Test of Research Instruments

No Item	Item Function	Validity test (Product moment)			Reliability Test (Cronbach's Alpha)		Difficulty Test		Question Differential Power Test	
		Value r_{count}	Value r_{table}	Description	Value	Description	Value	Description	Value	Description
1	Pre-test	0.739	0.355	Valid	0.649	Reliable	0.67	Currently	0.22	Enough
2		0.596	0.355	Valid			0.80	Easy	0.36	Enough

3		0.676	0.355	Valid			0.37	Currently	0.50	Good
4		0.594	0.355	Valid			0.33	Currently	0.22	Enough
5		0.665	0.355	Valid			0.35	Currently	0.28	Enough
6		0.631	0.355	Valid			0.56	Currently	0.22	Enough
7		0.667	0.355	Valid			0.80	Easy	0.39	Enough
8	Post-test	0.739	0.355	Valid	0.654	Reliable	0.38	Currently	0.42	Good
9		0.562	0.355	Valid			0.35	Currently	0.22	Enough
10		0.649	0.355	Valid			0.32	Currently	0.33	Enough

b) Instrument in the form of a questionnaire

The purpose of using a questionnaire instrument in this research is to measure the feasibility test of the module that has been developed, which is assessed from several aspects such as material, language and media which have been presented based on the grid guidelines in Table 3.

Table 3. Questionnaire Instrument Grid for Material, Language and Media Experts

No	Aspect	Assessment Indicators	Total Item
1	Material	Content Eligibility Feasibility of Presentation Straightforward	23
2	Language	Communicative and Interactive Suitability to Student Development Conformity to Language Rules Use of Terms, Symbols and Icons	10
3	Media	Module size Module cover design (cover) Module content design	22

Prior to research subjects using the modules to assess students' comprehension skills in the classroom, feasibility tests were conducted on modules that have been developed from multiple perspectives. Additionally, data exploration is done after the module is used to gauge how well students responded to it or whether it would be useful for instructors and students in the subject area. The details are shown in Table 4.

Table 4. Questionnaire for responses from students and teachers in the field of study

No	Aspect	Assessment Indicators	Total Item
1	Student Response	Interest Material Language	10
2	Lecturer Response	Interest Material Language	10

The data analysis techniques used are specifically presented in Table 5.

Table 5. Research Data Analysis Techniques

Research Purpose	ADDIE Model	Instrument	Data Acquisition	Measurement technique	Analysis Techniques
Development of Discrete Mathematics Modules based on Discovery Learning	Analysis (A)	Interview guidelines	Analysis of student and lecturer needs	Description	Qualitative descriptive
	Desain (D)	Documentation	Analysis of the study program curriculum	-	Descriptive
	Development	Questionnaire	Draft	Scoring:	Quantitative

Research Purpose	ADDIE Model	Instrument	Data Acquisition	Measurement technique	Analysis Techniques
	(D)		Module developed	- Very Valid: Score 5 - Valid: Score 4	descriptive
			Expert validation of material aspects	- Fairly Valid: Score 3 - Invalid: Score 2 - Invalid: Score 1	
			Expert validation of language aspects		
	Implementation (I)	Questionnaire	Expert validation of media aspects	Scoring: - Impractical: Score 1 - Less Practical: Score 2 - Fairly Practical: Score 3 - Practical: Score 4 - Very Practical: Score 5	Quantitative descriptive
			Student response		
	Evaluation (E)	Test	Lecturer (practitioner) response	Scoring: - Overall understanding: Score 4 - Partially understand: Score 3 - Some misconceptions: Score 2	Quantitative descriptive
			Pre-test and Post-test	- Misconceptions: Score 1 - Do not understand: Score 0	
Effectiveness of Discrete Mathematics Modules based on Discovery Learning for Mathematical Understanding	-	Test	- Post-test	Scoring: - Overall understanding: Score 4 - Partially understand: Score 3 - Some misconceptions: Score 2 - Misconceptions: Score 1 - Do not understand: Score 0	Quantitative descriptive

Results and Discussion

1) Development of a Discrete Mathematics Module based on Discovery Learning

In the module development process of Studying Southeast Asian countries in, there are 5 steps in development: 1) Content analysis by researching, assessing or defining 2) Designing the module 3) Implementing 4) Developing and revising 5) Evaluating (Sirisuthi & Chantarasombat, 2021; Daryono & Rochmadi, 2020; Efendi et al., 2019, February; Elmunsyah, Anggraeni, & Handayani, 2017, Kowitlawakul, Chan, Tan, Soong, & Chan,



2017, November; Fajaryati, Nurkhamid, Pranoto, & Muslikhin, 2016). Similar to the ADDIE model steps (Welty, 2007) used for discovery learning-based module development studies include the Analyze (A), Design (D), Develop (D), Implement (I), Evaluate (E) stages. Specifically explained as follows: Studying Southeast Asian countries in the module development process, there are 5 steps in development: 1) Content analysis by researching, assessing or defining 2) Designing the module 3) Implementing 4) Developing and revising 5) Evaluating (Sirisuthi & Chantarasombat, 2021; Daryono & Rochmadi, 2020; Efendi, Musnir, & Situmorang, 2019, February; Elmunsyah, Anggraeni, & Handayani, 2017, Kowitlawakul, Chan, Tan, Soong, & Chan, 2017, November; Fajaryati, Nurkhamid, Pranoto, & Muslikhin, 2016). Similar to the ADDIE model steps (Welty, 2007) used for discovery learning-based module development studies include the Analyze (A), Design (D), Develop (D), Implement (I), Evaluate (E) stages. Specifically explained as follows:

a) Stages of Product Requirements Analysis

The study of developing discovery learning-based modules to improve this ability starts from the analysis stage; as we know, mathematics is a science that studies abstract concepts carried out through calculation and measurement processes and expressed by numbers or symbols and has logical relationships (Amir, 2015). Considering the existence and urgency of HOTS, it is a challenge for education in the 21st century. The HOTS paradigm demands more complex thinking in facing and solving problems (Amir, 2020). So, students need to understand the concept first.

Furthermore, the needs and curriculum analysis results were carried out at UIN Syekh Ali Hasan Ahmad Addary Padangsidempuan as the first step in product development. New learning media requires analysis to determine the feasibility of implementing the learning media. Based on the results of the interviews conducted, the following factual narrative was obtained:

Respondent A said that:

"The reference books used do not attract students' interest in reading them because reference books still tend to only contain written formulas which make students confused and bored to study them, besides that the explanations are still incomplete, for example to solve problems related to discrete mathematics... .."

Followed by student respondent B who said that:

"The reference book used provides work or processing instructions that are unclear and difficult for students to understand. Apart from that, there are less real examples of discrete mathematics in everyday life....."

Followed by student respondent C who said that:

"The reference books used present material in language that is not easy to understand, making it difficult to understand the material. Apart from that, there is a lack of examples of questions in reference books in the context of discrete mathematics....."

The lecturer continued to say that:

"The reference books used present material in language that is not easy for students to understand, there are still not many examples of questions, especially for story questions because students still have difficulty understanding and converting them into discrete mathematical models, for example, the steps for solving sample questions are still unclear. so that students find it difficult to understand....."

Source: Interview Results (2023)

Meanwhile, analysis of the UIN Syekh Ali Hasan Ahmad Addary Padangsidempuan curriculum on the Mathematics Education study program. Media is developed according to the learning context, which directs students to be active. The material that will be developed in this media is a discrete mathematics course. This material is one of the materials contained in the basic competency standards for discrete mathematics lessons and must be achieved by students, one of which is through learning experiences. In the formal educational setting of discrete mathematics courses, students will encounter additional challenges. Mathematical problems are expressed as questions (Simamora & Saragih, 2019). Teaching should be

characterized not only by what instructors say and do but also by how teachers and students think and, most importantly, by the ways in which teachers apply students' thinking to their practice (Teuscher et al., 2015). To meet the needs of these students, teachers can use and develop modules according to the needs of existing problems in the class (Utomo et al., 2020).

b) Product Design Stage

This stage aims to describe the development design carried out in the research. This design stage includes three important parts in the module being developed, namely (1) Opening section; (2) Body/content section and (3) Closing section. Yerizon et al. (2018) state that the methods used to make students active are not enough, but teachers need to provide teaching materials to enable students to learn easily. Basically, learning modules must be designed by paying attention to communicative, constructive, presentational, pedagogical accuracy and language used to avoid abstract material (Lumbantoruan & Natalia, 2021). Bearing in mind, In mathematics, ideas are systematically arranged from the most basic to the most complex. The current ideas in mathematics education need to focus on conceptual understanding (Yuliani & Saragih, 2015). Thus, investigative activities, actions through the module design process to improve students' ability to understand concepts (Ba -Ader & Carlson, 2022; Utomo et al., 2020; Teuscher, Moore, & Carlson, 2015 ; Householder & Hailey, 2012).

c) Product Development Stages

The development stage is a stage for testing the validity of the module based on the assessment of material, language and media experts. Van den Akker et al. (2013) say that A good learning aid should satisfy the validity requirements by outlining the features of the product that is considered to be valid in order to represent the state of the art. In line with Nieveen & Folmer (2013), when a learning module satisfies the criteria of validity, practicality, and effectiveness, it can be considered feasible. The success of module development cannot be separated from contributions and a good development process (Sirisuthi & Chantarasombat, 2021). So, it is possible to continue at the implementation stage.

Meanwhile, the results of this feasibility test are used to assess products from various aspects such as material, language and media. Experts conducted feasibility tests on the development of modules based on discovery learning to assess students' comprehension of concepts shown in Figure 2. The results are listed below.

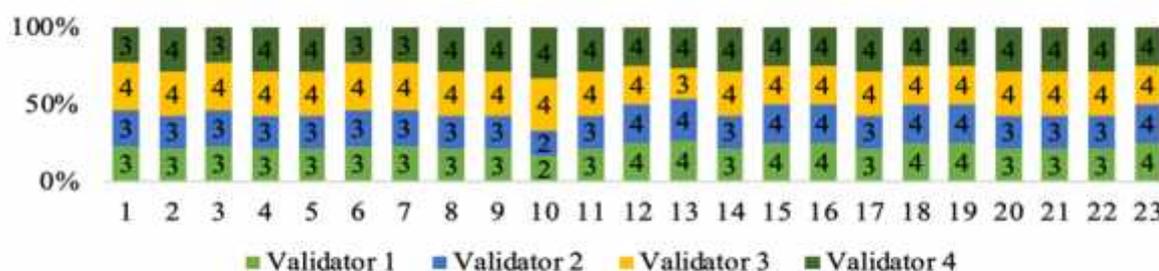


Figure 1. Results of Material Expert Validation of Module Development

Based on Figure 2, it can be interpreted that the expert assessment feasibility test consists of two stages. In the first stage, the assessment for validator 1 obtained an average of 3.26 (Quite valid) and validator 2 obtained 3.30 (Quite valid) with several suggestions and input from validator 2. After revisions were made to the development of this discovery learning-based discrete mathematics module Stage 2 assessment was carried out, with the average score obtained for validator 1 being 3.95, categorized as valid and validator 2, which was

3.82, categorized as valid. After that, a validity test was carried out in terms of language, as presented in Table 3.

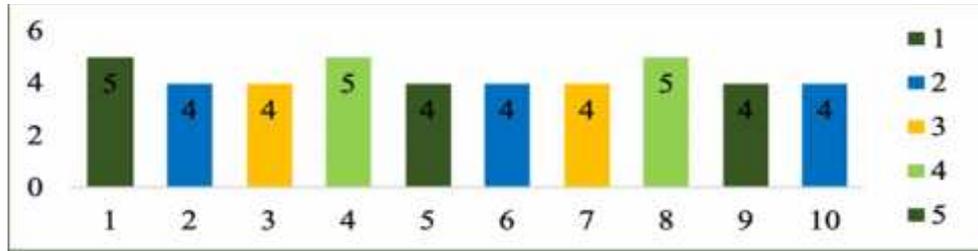


Figure 2. Linguistic Expert Validation Results on Module Development

Based on Figure 3, it can be interpreted that the theoretical feasibility test in terms of language obtained an average value of 4.3. It can be concluded that this value is included in the “Very Valid” category. After carrying out the validation stage by the language expert validator, suggestions were obtained from the validator. Suggestions given from the validator will be input for revising the product design regarding discovery learning-based mathematics modules with suggestions for improvement, choose standard words to use in (your) modules and improve typing. Meanwhile, feasibility testing was carried out in terms of media which is presented in Figure 4.

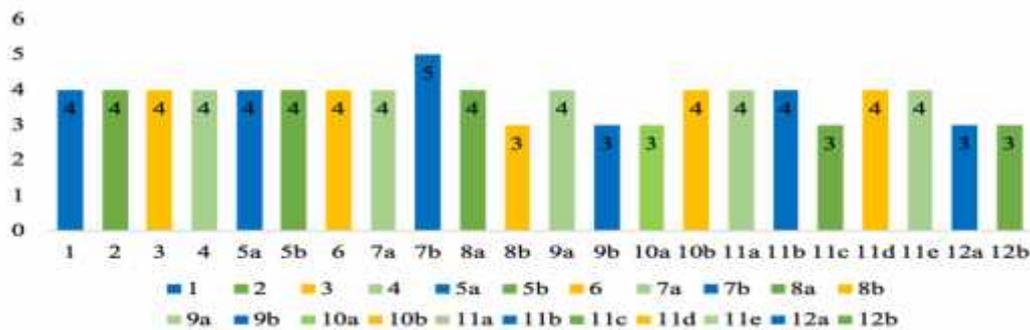


Figure 3. Media Expert Validation Results for Module Development

Based on Figure 4, the average language value = 3.77 can be concluded that this value is included in the valid category with suggestions for improvement from the validator. Thus, it can be concluded that the four validators’ expert feasibility test yielded an average total validity value of 3.96 for the module based on discovery learning. It can be concluded that the developed module based on discovery learning satisfies the validity requirements in the valid category.

d) Product Implementation Stages

It is advised that during this implementation phase, constructivism-based learning, such as discovery learning, be created with learning outcomes in mind (Minarni & Napitupulu, 2020; Arends, 2012). Bruner defines discovery learning as an instructional strategy that draws on constructivist inquiry-based learning theory and involves students solving problems by drawing on their prior knowledge and experiences to uncover connections and details about the newly studied material (Simamora & Saragih, 2019; Bruner, 1961). Coupled with stimulating learning module activities, easy-to-understand question language enables them to answer more accurately (Sirisuthi & Chantarasombat, 2021). It is in line with Hidayat et al. (2022) who state that meaningful teaching materials are about keeping the learning process interesting and not boring. According to field results, which are displayed in Figure 5, students’ comprehension of mathematical concepts increases when they are given pre-test questions prior to using the mathematics module based on discovery learning and post-test questions following the use of the module specifically.

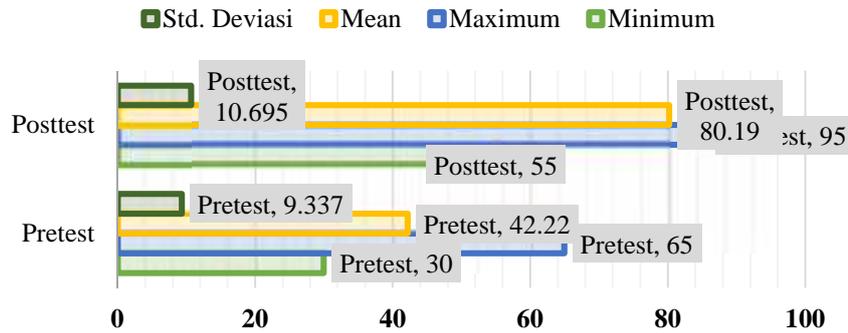


Figure 4. Increased Conceptual Understanding of the Use of Discrete Mathematics Modules Based on Discovery Learning

The statistical analysis of the data obtained indicates that the average pre-test score was 42.22 and the average post-test score was 80.19 (Score range 37,97), as summarized in Figure 5. In comparison to learning outcomes following the use of the discovery learning-based mathematics module, it indicates that there is an increase in learning outcomes prior to using the module. It demonstrates how the mathematics module based on discovery learning can enhance students' comprehension of mathematical ideas while they are in class. The discovery that the post-test average was higher than the pre-test came next. It means that there is an increase in learning outcomes before using the discovery learning-based mathematics module compared to learning outcomes after using the discovery learning-based mathematics module. It can be concluded that the results of the concept understanding test analysis are in the "Effective" category with an interpretation of 61% - 80%.

e) Product Evaluation Stage

This stage aims to determine the response of students and lecturer to the development of the module. Obtain data obtained in the field in the form of practical test data for using the module which is presented in Table 6.

Table 6. Student and Lecturer Responses

No	Respondent	Mean (%)
1	Students	82.04
2	Lecturer	80
	Average	81.02
	Category	Very Practical

Practicality is said to be practical because the modules that have been prepared can be used in the field with positive responses from students and teachers, as shown in the student response questionnaire that has been distributed, with a percentage of 82.04% (practical category). Meanwhile, judging from the response of teachers in the field of study (practitioner), the percentage could be 80% (practical) so that the average percentage of student responses and teacher responses was obtained at 81.02% at the interval 81%-100% in the "very practical" category.

After going through several stages, it is necessary to reflect to evaluate the activities that have been carried out, which show that there is a positive significance value for students' understanding of mathematical concepts during the process and use of modules using this discovery learning approach with an average pre-test score of (42.22) and the average post-test score is (80.19). The difference in average results is a relevant reference that this discovery learning-based module provides benefits to students' understanding of

mathematical concepts in class. It is in line with the research results of Yuliani & Saragih (2015), which showed that guided discovery learning has been shown to improve students' conceptual understanding and critical mathematical thinking. In line with research by Saragih & Afrianti (2012), which states that increasing students' comprehension of trigonometric function graph concepts will lead to a guided discovery method.

2) Effectiveness of Discrete Mathematics Modules based on Discovery Learning for Mathematical Understanding

The results of inferential statistical tests using the t-test to determine differences in average learning outcomes are presented in Table 7.

Table 7. Independent Sample t-test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
Value		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Value	Equal variances assumed	,039	,844	13,894	52	,000	-37,963	2,732	43,446	32,480
	Equal variances not assumed			13,894	51,070	,000	-37,963	2,732	43,448	32,478

Based on the results of calculation analysis using SPSS 24, the Independent Sample t-test significance value (sig. (2-tailed)) = 0.000. Based on the testing criteria, a significance value (sig. (2-tailed)) of $0.000 < 0.05$ was obtained, so it can be concluded that H_a was accepted and H_0 was rejected, meaning that there was a difference in the average test score of students' concept understanding abilities between the pretest and posttest. According to the findings of several earlier studies, an important area of empirical research on discovery learning in educational mathematics examines the process by which students make discoveries (Loibl & Leuders, 2018).

In previous empirical research, discovery-based learning focused more on evaluation, allowing students to struggle in the problem space before moving on to new material (Abrahamson & Kapur, 2018). So, this discovery learning-based module development study has positive benefits for students in the classroom, especially in increasing their understanding of mathematical concepts. Previous research also states that mathematics projects supported by focused instruction are key to guiding students to understand mathematics (Ferrill, 2017).

Jew (2012) adds that discovery learning can significantly increase material mastery, retention, and transfer of knowledge and learning. The research results of Hendri, et al. (2019) stat that discovery learning-based mathematics learning tools can make learning more practical and effective. Herdiana et al. (2017) also report research results showing that enhancing one's ability to solve mathematical problems through guided discovery learning is beneficial. It is consistent with Effendi's (2012) research findings, which demonstrate that when employing the guided discovery method, students have a positive attitude toward learning mathematics and are forced to think more exploratorily rather than merely mechanically and procedurally.

Through various series of activities also carried out by Park (2017), the experimental class exposed to the guided discovery learning model performed better on the final exam than the control group exposed to traditional instruction. It is demonstrated by the final exam results, which showed that students in the experimental class received an average score of



86.00 while those in the control class received an average score of 76.93. This research can be used to differentiate the quality of learning. Simamora & Saragih (2019) state that by including guidance in learning activities, discovery learning is developed to anticipate misconceptions or incomplete or disorganized knowledge, like creating modules to use as teaching aids in the classroom.

The development of quality modules facilitates teachers to guide students more effectively (Kiong et al., 2022). Students' and teachers' responses to our expert assessment indicate that the modules have been developed to be user-friendly and easy to understand. It is consistent with better student learning outcomes through the development of this module (Tamrongkunan & Tanitteerapan, 2020; De Freitas & Griffiths, 2008). An improvement in motivation, learning outcomes, behavior, and other areas can indicate whether a product, method, or learning model is effective or not (Kariman et al., 2019).

Conclusion

The results conclude that (1) Reviewing the study of discrete mathematics module development obtained valid results after going through several assessments and revisions, and the practicality of developing this discrete mathematics module was categorized as practical, that the module can increase students' mathematical understanding with a range of 37.97; (2) Judging from effectiveness; it is obtained from inferential statistics that significance is 0.00, which means there is an influence on the use of discovery learning-based discrete mathematics modules on students' mathematical understanding. These results have implications that the module has proved effective in improving students' mathematical understanding in higher education.

Recommendation

This research provides concrete implications for lecturers in using discrete mathematics modules, the validity of which we have tested empirically, in order to make it easier in the field of teaching discrete mathematics in the classroom. Apart from that, concrete implications for students from using these modules can provide significant Mathematical Understanding in an area lecture on Discrete Mathematics in class. Thus, this research is a role model that produces outcomes that have an impact on universities everywhere.

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