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Realistic Mathematics Education (RME)-Based Learning Trajectory for Arithmetic Social Using Culinary Context of Yogyakarta

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Abstract: This study aims to analyze a learning trajectory for social arithmetic using the culinary context of Yogyakarta and the results of the retrospective analysis as an application of the hypothetical learning trajectory. The research method used was design research with three stages, namely preliminary design, teaching experiment, and retrospective analysis. The research subjects who contributed to this study were sixth elementary school graders (11-12 years old) using the purposive sampling technique. The data collection techniques were documents (written data on students' work), and participating-interview about students' thinking in mathematics during the learning process. Data analysis used retrospective analysis. The findings of this study indicate that this learning trajectory is beneficial in achieving learning objectives. The learning trajectory in this study can be considered as an alternative way or as a reference for teachers to design RME-based learning that supports the development of students' conceptual understanding, self-confidence using their strategies, communication, and reasoning skills about social arithmetic concepts. In addition, the learning process and the results of the retrospective analysis contain characteristics of Realistic Mathematics Education (RME).

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Introduction

Mathematics has an important role in human life. This is because mathematics is a basic and essential knowledge of science and technology (Raj Acharya, 2017). Humans use mathematics in various social, economic, engineering, medical, and other fields. The importance of mathematics is marked by using mathematics as one of the test elements in the PISA (Program for International Student Assessment) which is an international assessment as a benchmark for education in the world. However, in reality, mathematics is often experienced as difficult (Simmers, 2011). According to PISA results, one of the causes of these assumptions and feelings of difficulty is that students' ability to solve and interpret mathematical problems is still in the low category (Stacey, 2011; Wijaya et al., 2014). In order to change this assumption, a change in the learning process is needed, by designing a learning environment that is fun and concept relates to everyday life so that students can feel the benefits, meaning, and application of the lesson.

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The development and application of mathematics problems based on everyday life context is a part of the students' mathematics learning process (Madani et al., 2018; Tanujaya et al., 2017). Students' daily activities can be used to build mathematical knowledge, which can then be used as a starting point for learning. Lappan and Briars explain that selecting a task or activity is a significant decision in influencing the learning process (Simon & Tzur, 2004). According to Freudenthal (1999), tasks or activities in mathematics must support to re-inventing mathematical concepts, this is in accordance with the constructivist theory which assumes that students must build their own knowledge. Simon (1995) coined the term "hypothetical learning trajectory" (HLT), which refers to selecting tasks or activities by paying attention to what they focus on, imagining students' mental activities, and anticipating how their thinking can help them develop mathematical concepts that are targeted (Koeno Gravemeijer, 2015). In short, the hypothetical learning trajectory is an assumption of expected student activities (Weber & Lockwood, 2014). The HLT will go through stages until finally a learning trajectory (LT) is formed. LT is very helpful to bridge the work of researchers and practitioners. LT can also help teachers evaluate and rethink appropriate learning methods or strategies before entering the classroom. Many researchers have carried out research on the development of student learning trajectories to minimize or be a solution to students' learning obstacles (Cárcamo et al., 2021; Risdiyanti et al., 2019; Risdiyanti & Indra Prahmana, 2020; Wijaya et al., 2021). This provides an opportunity to conduct learning by relating lessons to the surrounding context or the closest context of students.

Freudenthal explained that mathematics is a human activity and must relate to everyday life (Freudenthal, 1999). However, in reality, the implementation of mathematics learning in schools tends to be taught using practical formulas (Arisetyawan et al., 2014), so that students feel less connected to the use of the concept with real-world problems. Therefore, it is important to design contextual and realistic learning (which can be linked to the real world) and learning experiences in the form of activities, these activities can be in the form of physical activities or mental activities in order to develop the ability to solve real problems.

RME (Realistic Mathematics Education) is one of the right approaches to answer the above problems. RME combines perspectives on mathematics, how students learn, and how mathematics should be taught (Hadi, 2018). Learning that makes a real situation as the starting point of learning is a feature of the Realistic Mathematics Education (RME) approach (Juandi et al., 2022; van den Heuvel-Panhuizen, 1996). Hans Freudenthal interprets the real or realistic meaning of RME, he has two main views which were mathematics must be close to children and relevant to everyday life situations. However, the word 'realistic', refers not only to the relationship with the real world, but also to real problem situations in students' minds (Hadi, 2018; van den Heuvel-Panhuizen, 1996; Zulkardi, 1999). The problem (task) to be presented to students means that the context can be real world (daily life context) but this is only sometimes necessary. Therefore, the word "realize" emphasizes that the context in RME learning can come from real things done or observed by students (enactive) or realized as real things even though they only exist in their minds.

One of several contexts that can be used in RME is a culture that is realistically applied in learning mathematics and adapts to the local context in which the school is located (Koeno Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen, 2005). As a result, the Yogyakarta culinary context can be used to teach students mathematical concepts, particularly social arithmetic. It will have a good impact on the enthusiasm and interest of students, it can also develop students' problem-solving abilities for daily-life problems (Sugiman & Kusumah, 2010).

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Mathematization through context can occur at different levels. These levels of mathematization are connected to the various levels of understanding through which students can pass: from the ability to invent informal context-related solutions to the creation of various levels of shortcuts and schematization to the acquisition of insight into underlying concepts. The essence of this research is to explore and describe students' activities and students' ways of thinking in solving math problems. Mathematics doesn't just about calculating and finding the right or wrong solutions. but also pay attention to how to get the solution (mental activity), and how student schemes are formed. This can be an evaluation for educators to create, plan, or apply a learning trajectory that can bring rich outcomes to students' progress. It is not only quantity (score of mathematics outcomes) but also the quality of the mathematics learning process.

Research Method

Design research was used as a research method in this study. Richey and Nelson, as well as Van den Akker, distinguish between two types of design research. Plomp and Nieveen (2009) classify these as development and validation studies. Researchers used design validation research using the design proposed by K Gravemeijer & Cobb (2006) because the study aims to develop a theory about the learning process of certain concepts and instructional designs to support such learning. This design research consists of three stages, namely the process of preliminary design (preparation for the experiment), teaching experiment (implementation of learning process), and retrospective analysis.

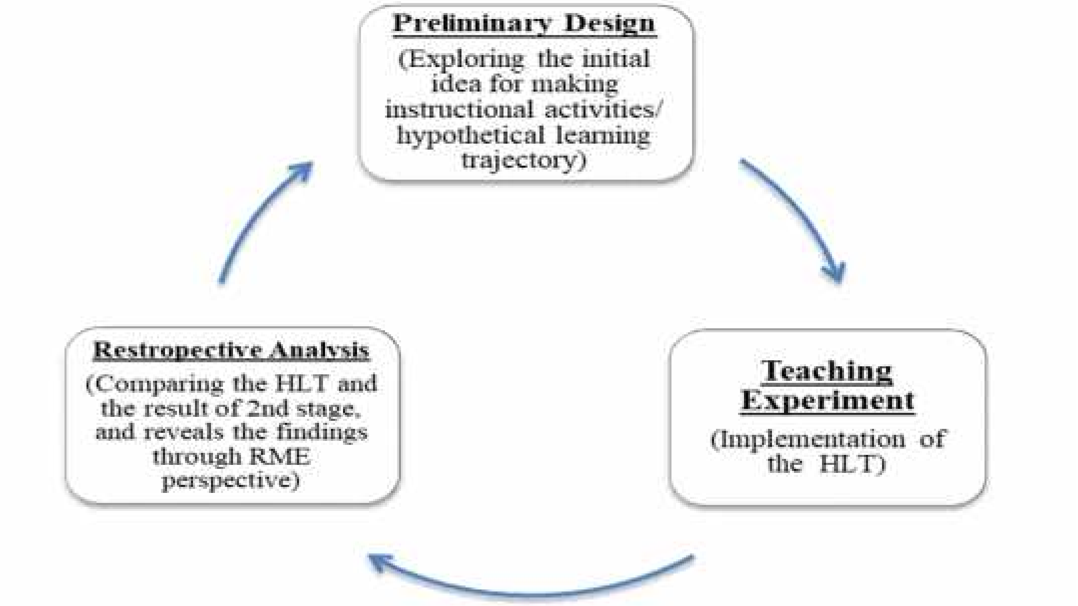

Figure 1. Research Stages

In the preliminary design stage, the initial idea for sequences of instructional activities was explored through reviewing the literature on social arithmetic and realistic mathematics education, observing students to find out students' learning obstacles, and interviewing teachers. The literature review aims to see the theoretical perspective. At this stage, a series of learning activities containing assumptions about student strategies and thinking was developed (HLT) (K Gravemeijer & Cobb, 2006).

In the second stage, learning (teaching experiment) was implemented by involving the research subjects from elementary school. It was done to investigate how the hypothetical

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learning trajectory can facilitate students in mathematics. The students' thoughts were observed in the learning process by involving participating-interview during the learning process. In the third stage, Analyze the data using retrospective analysis. All data collected in the experimental design will be analyzed by comparing the predictions in the HLT with the results of the implementation of the learning trajectory that has been carried out in the experimental design (teaching experiment) phase.

For the teaching experiment phase, six students of 6th graders elementary school (11-12 years old) participated in this study. Subjects were taken using purposive sampling. In addition, a mathematics teacher also contributes to providing information about students' learning obstacles. Data collection techniques used documents (obtained from the results of students' work), and participating interview conducted during the learning process. The data analysis used in the design research is a retrospective analysis (K Gravemeijer & Cobb, 2006), which is carried out by comparing the hypothetical learning trajectory (HLT) with real conditions at the teaching experiment stage.

Results and Discussion

Doorman in (Wijaya, 2008) explains that the results of design research are not the design itself but the underlying principles that explain how students' mathematical processes are and why this design can be a learning trajectory. The use of the Learning Trajectory is multidimensional; it makes many aspects contain variations. Some aspects found are LT can have different learning objects, LT can have different learning subjects, LT can be based on different theoretical perspectives, and LT can also vary in the scale of topic (Lobato & Walters, 2017). LT can have different learning objects, for example, elements of LT can be anything, cognitive conceptions (e.g., Battista, 2004) , forms of discourse (e.g.,Jin & Anderson, 2012), observable strategies (e.g.,Vermont Mathematics Partnership's Ongoing Assessment Project, n.d.), or textbook tasks (e.g.,Wang et al., 2017). LT can have different learning subjects, LT can focus on individual learning (e.g.,Steffe, 2004) and mathematics in the collective classroom (e.g.,Cobb et al., 2003). LT can be based on different theoretical perspectives such as Piagetian schemes and operations (e.g.,Hunt et al., 2016), hierarchic interactionalism (e.g.,Clements & Sarama, 2014; Cobb & Yackel, 1996), emergent perspective (Stephan & Akyuz, 2012). LT can also vary in the scale of topic selection, LT can be used to deal with single concepts (e.g., partitive reasoning in fractions (Norton & Wilkins, 2010))) or can cover multiple topics and grade levels (Smith et al., 2006). Below (see table 1.) are the details of the Hypothetical Learning Trajectory (HLT) used in this study.

Table 1. Hypothetical Learning Trajectory

Activity	Sub Topic (Learning Goals)	Hypothesis
Task 1. Determining the purchase price of Lumpia samijaya	Produce Pricing & Many Changes	• Students can determine the quantity of Lumpia that they got, if they pay a certain amount of money. • Students can determine the price to be paid if they buy in a certain quantity (different types of Lumpia). • Students can determine the price to be paid if they buy in a certain quantity and the change they get if they pay in a certain amount of money.

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Task 2. Analyzing discount ads in the Gudeg store	Discount	• Students can interpret the concept of discount. • Students can determine the sale price after getting a discount.
Task 3. Analyze the buying price and selling price of Bakpia Tugu.	Profit & Loss	• Students can interpret the concept of profit and loss by comparing the selling price and buying price of Bakpia Tugu.

The results and discussion in this study concern the development of students' ideas and answers toward activities (tasks) that have been designed to support learning. Descriptions of the above developments can be seen in each activity sub-theme.

Task 1. Determining the Purchase Price of Lumpia Samijaya

In the first activity, students determine the quantity of Lumpia they get if they pay a certain price, students determine the purchase price of Lumpia according to the specified variant, and the number of people who will be given (See Figure 2.)

On Sunday, Ela and Rina decided to take a walk exploring the street food on Malioboro Street. Malioboro Street is one of the famous streets as a tourist center. After some time thinking about which snacks to buy, they finally bought one of the legendary dishes, namely "Lumpia Samijaya". With prices listed below:

Types of Lumpia	Price
Lumpia Special (Chicken, Vegetables, boiled quail egg)	Rp6.000,00
Chicken Lumpia (Chicken, Vegetables)	Rp5.000,00

Instructions:

a) Ela brings Rp. 30,000.00 and wants to spend all of it to buy Lumpia. Write down the probability of how many Lumpia Ela will buy with that money and the types of lumpia that she might choose, explain your answer!
b) Rina wants to buy 5 Chicken Lumpia and 10 Lumpia Special for herself, how much money should Rina pay? write down your explanation or strategies!
c) Rina then thought that she would buy for her family of 6 people at home, 1 person she would give 1 chicken Lumpia and 1 special Lumpia. What price do you think Rina will pay? If Rina pays with IDR 100,000, how much change does Rina get? Explain your answer, you can draw or write anything that helps you answer this question!

Figure 2. Sample of the First Activity

The results of the researchers' observations on the progress and development of students when carrying out task 1 were quite good because students were able to achieve the objectives of learning. However, some students have not been able to communicate well in writing the strategies they used. Student A can determine the number of Lumpia if they are purchased at a certain price (Figure 3). It is seen that the students know how to determine it, but they still need to improve their communication skills in writing.

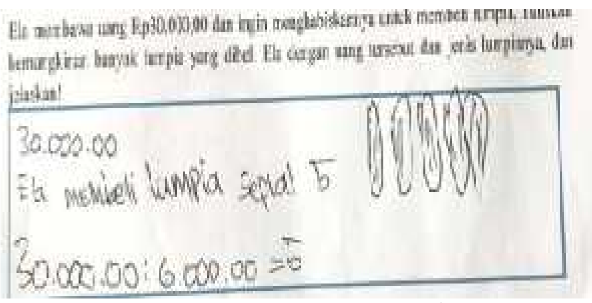

Figure 3 : Student A's Work on 1st activity (a)

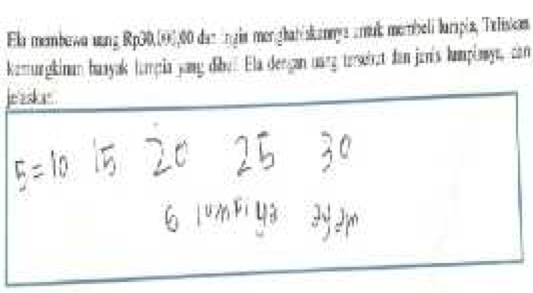

Figure 4 : Student B's answer on 1(a)

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Student A: *"If it's IDR 30,000, it means that there are two ways, right, ms?"*
Researcher: *"Oh yeah? How can you think about that?"*
Student A: *"30 is a multiple of 5 and 6 Ms., if I only use the special Lumpia, if the price is Rp. 6,000.00 for each, it means there are 5 Special Lumpia."*
Researcher:" *Why 5?"*
Student A: *"yes, because 6 times 5, 30 Ms."*

Student A uses the formal method by directly dividing the amount of money by the price of each Lumpia. Meanwhile, student B uses the informal method. This shows the achievement of one of the characteristics of RME, namely students using their own construction and productions and Using models and symbols for progressive mathematization.

Student A knows that there are two answer solutions. That student determines the quantity of special Lumpia purchased for Rp. 30,000.00. While student B answered it by writing multiples of 5 to 30 so that he could get an answer of 6 chicken Lumpia for Rp. 30,000.00. Good mathematics achievement can be seen in the variety of students' mathematical ideas and this can be supported by displaying tasks that contain open-ended questions (Pehkonen, 1997).

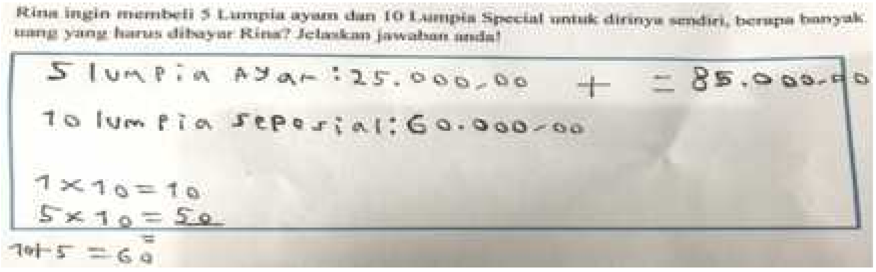

Figure 5. Student B's answer on 1(b)

Student B: *"If there are 5 chicken Lumpia, IDR 5,000 for each, it means 25,000, Ms. But if 10 special Lumpia Rp. 6,000.0 for each, that means 10 times 6, right? Hmm (thinking) Oh I can separate 5 and 1 so there are 10 5s and add up with 10?"*
Researcher:" *yes it makes sense!"*

(See figure 5.) Student B can do well with multiplication by 5 so it was easy to determine 5 Chicken Lumpia for Rp. 5000.00/lumpia, in determining special Lumpia as many as 10. That student finds it difficult to multiply 6 and 10, this student involves the concept of distributive properties in operations by separating 5 and 1 in 6, therefore $6 \times 10 = (5 \times 10) + (1 \times 10)$. The use of the concept of multiples, multiplication, and properties in operations indicates that the development process and student answers involve combining several concepts or *Intertwinment* which is one of the characteristics of RME. On the third question in the first task Students were asked to determine the price of Lumpia if it is known that the spring rolls will be given to 6 people with each person getting 2 Lumpias consisting of special Lumpia and chicken lumpia.

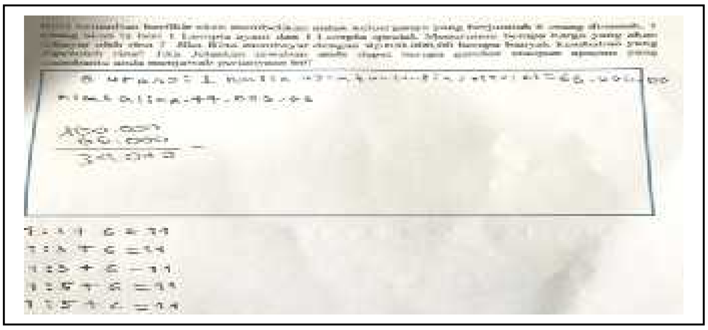

Figure 6. Sample of Student Answer on 1(c)

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(See figure 6.) The student answered by adding up the prices for each person who got 1 special lumpia and 1 chicken lumpia so that 1 person will be given 2 types of Lumpia which have a price of Rp. 11,000.00 so that the final answer for 6 people is Rp. 66,000. Furthermore, the students determine the change that will be received if they pay a certain nominal. By using their own method, students can determine the concepts of Produce pricing and Many Changes. Although in answering the questions in writing, students are still not proficient in written mathematical communication.

Task 2. Analyzing Discount Ads in the Gudeg Store

From this activity, students were asked to analyze the statement and determine what strategy was used to determine the savings from the original Gudeg price, and determine the sale price paid after the discount was applied.

Figure 7. Sample of Second Activity

The researcher found that good progress and development were seen in the learning process because previously students knew the definition of a discount, which is a reduction of savings from the original price. A description of the results of student answers can be seen below.

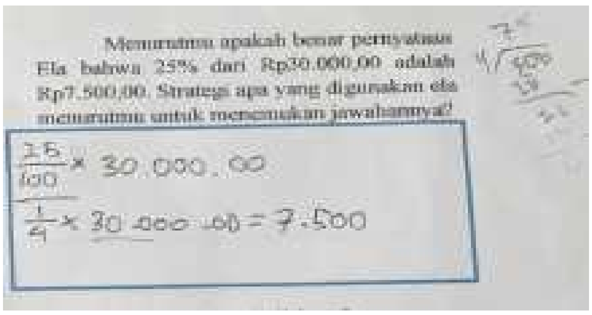

Figure 8. Sample answer of Student C

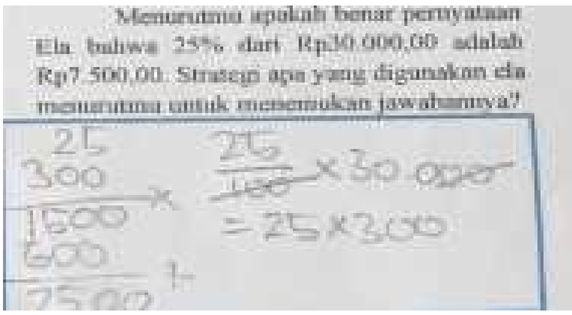

Figure 9.Sample Answer of Student A

Student C and student A determine the answer using their own way. Student C by simplifying the form 25% into $\frac{1}{4}$, meanwhile student A immediately multiplied 25% with the original price of the Gudeg to get the savings. Because students already know well the meaning of the discount for the answer to the next point, students immediately subtract the savings from the original price so that they can get the sale price to be paid.

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Task 3. Analyze the buying price and selling price of Bakpia Tugu.

Task 3 aims to determine the concept of profit and loss in a sale. Below is an example of a task and a sample of the progress and development of students in answering this task.

Bakpia Tugu is one of the special snacks of Jogja. Mr. Ardi is one of the Tugu Bakpia sellers who is a second hand (distributor). Mr. Ardi sells his Bakpia Rp 35.000 for each variant. On the last day of the week he counts his sales.

Variant	Qty	Buying Price (Rp/Pack)	Selling Price (Rp/Pack)	Income
Ori-Coklat	15	Rp20.000	Rp35.000	Rp350.000
Ori-Keju	20	Rp30.000		Rp525.000

Mr. Ardi also noted that he still has 5 packs in each variant that have not been sold, and they cannot be sold anymore because they have expired.

Instructions:

a) How much is the total buying price paid by Mr. Ardi for each variant from the information you get in the table? explain your answer!
b) What is the selling price that Mr. Ardi gets for each variant if there are still 5 packs of each variant that have not been sold? Explain your answer!
c) Compare the selling price and the purchase price for each bakpia variant, and explain what you think about Mr. Ardi's income? Explain your answer!

Figure 10. Sample of Third Activity

The results of the researchers' observations found that students began to find it difficult when they encountered multiplication in a more complex form but could still be overcome by students. In this task, students can determine the concept of profit and loss well.

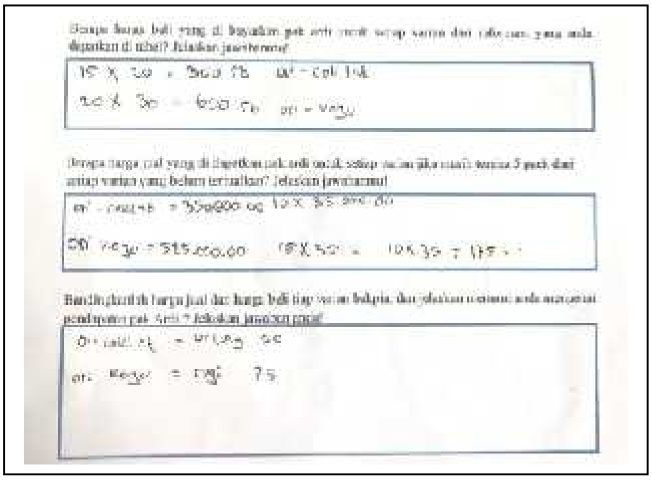

Figure 11. Sample answer of Student D

Student D: "*The top one (ori-chocolate) was sold for Rp. 350,000.00 which means profit, right, Ms*"
Researcher: "*Why profit*"
Student *D: "yes? if we buy it for Rp. 300,000 and sell it there is a profit of Rp.50,000?"*
Researcher: "*Yes, how about the bottom one (pointing on ori-keju)?"*

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Student D: “*hmm (thinking) it’s loss, ms?*”
Researcher: “*how much?*”
Student D: “*Wait a minute, I’ll look for it first.*”

Basically, students in daily life understand the meaning of loss and profit, it is simpler to say profit if it is more than the initial price, and loss if it is less than the initial price.
The form of student answers above (task 1 – task 3) involves the principles and characteristics of RME in answering them. The problems come from real problems (Didactical Phenomenology) and were used as starting points that bridge students' knowledge to find social arithmetic concepts under the guidance of the teacher and task instructions (guided reinvention) (Freudenthal, 1999; K. P. E. Gravemeijer, 1994). Besides, the task provides opportunities for students to develop strategies that are used following the characteristics of the RME, namely, using students' construction and productions and completing their models. Using models and symbols for progressive (Van den Heuvel-Panhuizen & Drijvers, 2020). The answer involves combining the concept or intertwinment, implemented with an interactivity process. And the achievement of all learning objectives that have been hypothesized previously.

The formed Learning Trajectory includes the initial concept model and operation, an explanation of the development in students' mental activities and operations observed through interactive activities with students, and an explanation of the mathematical interactions involved in the learning process. Often students try to ignore or avoid using their own spontaneous way if there is no relationship between students' prior knowledge and the concept being taught, this is usually done with the aim of a tendency to adjust to the "formal" class (Hiebert, 1984).

In learning mathematics, students follow natural developments in the learning process. Students use and develop mathematical ideas and skills in their own way(Clements & Sarama, 2014). The process is through physical and mental activities of students during the learning process. The ability of students to find, learn, apply, justify, and reason about arithmetic shows that the use of algebraic thinking skills plays an implicit but significant role (Clements & Sarama, 2014; Dian & Max, 2011). In elementary school students can generalize and use symbols to represent mathematical ideas, represent problems, and solve problems. For example, students draw when answering questions involving the concepts of the basic properties of addition, subtraction, multiplication, division, and also the relation between operations. The students use commutative addition to create a “counting-on-from-larger” strategy. They also use the concept of distributive properties to help multiplication that students have not mastered and use simplification in fractions to make it easy to operate.

In qualitative research, results vary greatly depending on the results found in the research field. It is very pleasant that this learning trajectory can develop their abilities and bridge students in discovering social arithmetic concepts. The study describes students' physical and mental activities in the learning process from the principles and characteristics of RME. Interactions on how students think and finally found a solution to the problem is important. As Freudenthal (1999) mentioned that what humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics.

Conclusion

The conclusion obtained from the results of this study is this learning trajectory is very helpful in achieving learning objectives. The learning trajectory in this study is an alternative

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method or frame of reference for teachers in designing a sequence of Realistic Mathematics Education-based learning activities which support the development of students' conceptual understanding, self-confidence, communication skills, and reasoning about the social arithmetic concept.

Recommendation

Some recommendations for teachers as educators are to bring the context of mathematics and pay attention to providing scaffolding to students to bridge students' informal knowledge to formal knowledge. provide time and rich context for students to be able to develop their strategies through the given task or learning trajectory. Instead of being receivers of ready-made mathematics, the students are treated as active participants in the educational process, developing all sorts of mathematical tools and insights.

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