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Problem Decomposition Skills, Mathematical Maturity, and Their Relation to Mathematics Problem-Solving in A Computer Science Learning Class

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Abstract: This study investigates how students represent ideas when decomposing mathematical problems and how their mathematical maturity influences the problem-solving process. The method used in this research is explorative research. The subject of this research was six Computers Science Education Department students at the Indonesian Education University. The instrument used task-based interviews. Data analysis used the concept of Miles and Huberman, including data reduction, presentation, and drawing conclusions. The research found that problem decomposition skills, mathematical maturity, and their relation to solving mathematical problems in computer science learning classes influenced one another. Decomposition skills were influenced by how basic math skills are taught, so they can affect students' maturity in solving math problems.

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Introduction

Problem-solving skills have long been an essential orientation in mathematics teaching and learning. Problem-solving skills can be defined as one's willingness and ability to find a solution to a problem to obtain knowledge and understanding of the scientific concept of higher-order thinking skills. Mathematics teaching and learning assume that problem-solving skills are vital for developing disciplines and nurturing students' thinking processes (Leiljedahl et al., 2014; Polya, 1945; Usanova et al., 2016).

The tendency of students to memorize lessons holds a unique place in mathematics learning (Moursund, 2016). Most learning processes based on remembering materials tend to cause problems on some level, such as the student's inability to apply what they have learned to new situations both within and outside of the mathematics discipline (Moursund, 2016).That is because memorizing is relatively limited regarding long-term retention (Moursund, 2016). This condition impacts students in the form of a tendency to be slow and inaccurate in solving mathematical problems, which under certain conditions even tend to be impulsive (Gersten et al., 2011; Molnár & Csapó, 2018).

This situation is caused by low cognitive maturity (Buckle, 2018). According to Suharnan (Rahman & Saleh Ahmar, 2016), it could be the inhibiting factor that might occur in the problem-solving process, which includes understanding the problems presented and mental representations in solving problems. Cognitive maturity itself is the result of knowledge that is used effectively and involves certain mental activities (Buckle, 2018; Buitrago-Flórez et al., 2021) , such as the ability to respond appropriately to a cognitive situation, effective decision-making, and the context; knowledge flexibility; as well as the ability to consider various possible and relevant options in problem-solving efforts.

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(Moursund, 2016) uses the term "mathematical maturity" to describe the condition of knowledge, skills, insight, ways, and habits of thinking about mathematics in a person. The existence of mathematical maturity in a person is an indication of one's ability to see the "big picture" of a concept as well as a description of one's abstraction ability in grouping an idea, internalizing it, and making it a reference in constructing new understandings (Krantz, 2012).

It is then interesting to study that mathematics occupies its own position in computer science with the development of current technology. It is very understandable if computer science is significantly different from math in terms of the visibility and ease of use of some of the results of its scientific novelty (Moursund, 2007). Thus, it is natural for those working in computational science to have a strong, if not mature, foundation in mathematics (Moursund, 2007) to properly and successfully take advantage of math as a computational tool in a computer program.

The literature review reveals that computational thinking ability and mathematical ability/intelligence share several characteristics in their definitions. The two concepts are described as an approach and problem-solving ability to solve problems in any context. Computational thinking ability is often referred to as the process of solving problems with a computational approach, such as the ability to do abstraction and recognize patterns, the ability to describe problems or decompose and handle data, and the ability to design and implement algorithms (Seehorn, et al., 2011)(Csizmadia, et al., 2015)(Namukasa, Patel, & Miller, 2017) (Fernández, Zúñiga, Rosas, & Guerrero, 2018) (Boom, Bower, Arguel, Siemon, & Scholkmann, 2018)(Rodríguez del Rey et al., 2020). On the other hand, in his classic work on problem-solving abilities, (Polya, 1945) explains that abstraction abilities, which are defined as a combination of analogy, generalization, and specialization with decomposition skills, are the main obstacles for students in solving cases related to problem-solving abilities (Barcelos et al., 2018)

Ideally, the computer science curriculum must be directed toward developing the power of abstraction and the ability to find abstractions suitable in certain situations (Scherlis & Shaw, 1983), as well as the skill of problem decomposition (Laski et al., 2014). This is the skill that mathematicians call mathematical maturity. Mathematical maturity will not be achieved if math is only taught as a computational skill (Scherlis & Shaw, 1983), especially in computer science.

Mathematical maturity comes with an implicit understanding that mathematical problem-solving is regulated by one's ability to negotiate specific parameters based on axioms, theorems, guesses, etc. However, mathematical maturity allows for the ability to reassess mathematical properties and their use so that it is possible to reapply, create, and reclassify them (Soutter & McKubre, 2018) (Braun, 2019). Mathematical maturity talks about the method of proof and the methods of thought needed to build and interpret evidence (Ponomarenko, 2019). The ability to perform abstraction and solve problems is the determining skill of mathematical maturity. In his classic work about problem-solving abilities (Polya, 1945) explains that abstraction skill, defined as a combination of analogy, generalization, and specialization along with decomposition skill, is the main obstacle for students in finding solutions to problem-solving-related cases (Barcelos et al., 2018).

Decomposition has a strategic position in computer science because it is the basis of various computer science activities (Kotu, V., & Deshpande, 2019; Mo, J. P., Bil, C., & Sinha, 2015; Mukhin et al., 2020). For example, the study deals with architectural synthesis problems in modern computer systems that control complex distributed objects (CDOs). This problem can only be solved by involving decomposition, structuring, and formalization processes (Mukhin et al., 2020). Besides, decomposition is one of the components of computational thinking (Wing, 2010). In the field of mathematics itself, problem decomposition has become an issue that is studied in depth in order to perfect various efforts to determine solutions to various mathematical problems, for example, to find optimal partitions (subsystems) of a particular statistical system (Tuncer et al., 2008); large linear programming problems with special structures ; matrix linear algebra (Borndörfer et al., 1997), and so on.

This research discusses problem decomposition skills, mathematical maturity, and their relation to mathematics problem-solving in a computer science learning class. Decomposition in mathematics is considered an ability that depicts the depth of students' understanding of the problem (Laski et al., 2014). Decomposition not only has an impact in the form of computation strategy but also improves performance in mathematics (Laski et al., 2014). The decomposition skill effectively assists problem-solving processes (Koopman, 1995). There is a fact that states that human memory is limited to 7±2 items at a particular time, which means that some problems are too complicated to be solved by the human brain unless the problem is solved first and broken down into subproblems to be processed in the brain (Labusch et al., 2019). The process of breaking down a problem into smaller parts to be more easily managed is known as decomposition (Barcelos et al., 2018). Decomposing the problem by analyzing it as a whole requires knowledge of the manageable steps and how they are related to one another to form the main problem (Labusch et al., 2019).

Decomposition is magic that allows us to solve complicated problems (Krauss & Prottsman, 2017). To put it simply, decomposition breaks down problems into smaller, easier-to-solve parts. Problem-solving is the only thing that can help reveal the solutions to a given problem. It will also be simpler to see a problem in smaller pieces rather than as a whole (Krauss & Prottsman, 2017). The focus of the decomposition process may vary depending on the type of problem that will be decomposed. Decomposition can be related to matter, processes, and mental states, forming the three main ontologies for reasoning and learning (Chi et al., 1994; Koopman, 1995). Koopman (1995) prefers to use the terms "structures," "behaviors," and "goals." "Structures" represent physical components, logical objects, or geometrical attributes. Meanwhile, "behaviors" tend to describe the presence of data transformation and cause-effect relations, among other things. On the other hand, "goals" refers to targets, costs, and aesthetics (Koopman, 1995).

In mathematics, the decomposition process is used not only as a strategy for parsing a mathematical concept for learning purposes but also for teaching itself. Decomposition teaches that the essential part of the professional teaching process is identifying learning components that are integral to learning as a whole and increasing the processing capacity of these components through specific/targeted instruction (Arnon et al., 2014)(Sztjan et al., 2020). Knowledge grows based on specific mechanisms. It includes three levels (intra-level, inter-level, and trans-level), which develop in a fixed order and are called the "triad" (Dubinsky, 1991). The fixed order means that the triad level is hierarchical, with the intra-level as the lowest, the inter-level as the middle, and the trans-level as the highest. Another characteristic of the triad level is that it is functional, not structural. Thus, when one faces a problem, their schema does not have to develop from the lowest level. The scheme of development to solve the given problems will be mapped to one of the triad levels.

Some researchers use the term "cognitive path" to describe the sequence of concepts students demonstrate when studying a particular math topic or solving math problems (Arnon et al., 2014). However, (Arnon et al., 2014) distinguish between the two of them—cognitive path and genetic decomposition. The cognitive path describes the students' cognitive development as being linear. Meanwhile, genetic decomposition opens up the possibility of building in a variety of ways (Arnon et al., 2014).

Besides explaining how concepts can be constructed mentally, decomposition also makes it possible to present prerequisite structures that the students must build and even explain the differences in students' development due to the variations in mathematical performance (Arnon et al., 2014). Therefore, it can be said that decomposition is an epistemology and cognition model of a particular mathematical concept (Arnon et al., 2014; Roa-Fuentes & Oktaç, 2012). Despite individual differences, decomposition illustrates the structures students must build to learn a particular concept. In addition, when verified empirically, decomposition functions as a cognitive model to describe the students' conception of a mathematical concept. Also, it can be used in effective teaching designs (Waller, K., Clark, J. M., Dubinsky, E., Loch, S., McDonald, M., & Merkovsky, 2003). (Egidi, 2011) defines this condition as an effort to solve problems by breaking down problems into sub-problems and optimizing the solution-seeking process of each sub-problem. The global solutions students obtain are generally suboptimal, depending on their decomposition patterns (Egidi, 2011).

Research Method

The method used in this research was explorative research. Exploratory research determines the nature of symptoms caused by the research subject (Siegel et al., 2003; Zhai et al., 2017). The problem or symptom is specifically examined in depth. The same thing was expressed (Morgan DL, 2000): carefully planned discussions produce a perception of a particular area of interest. The subject of this research were six Computer Science Education Department students at Universitas Pendidikan Indonesia —out of the 134 who participated in the research project. Two have vocational school backgrounds majoring in Computer Technology, while the others were generally high school students. Students with general high school backgrounds will be called Subject A and Subject B for research purposes. In contrast, those with vocational high school backgrounds will be called Subjects C, D, E, and F. The Computational Thinking Paradigm is an introductory course of the department intended to help students understand how computers should be utilized, mainly when associated with the role of the computer as a tool to solve problems. Students taking this course have passed introductory computer science courses, such as Introduction to Mathematics, Introduction to Programming, and Calculus, which are expected to lay an adequate scientific foundation to perform decomposition toward a given problem.

The instrument used task-based interviews. The task presentation generally followed a five-step plan: presenting the problem, having the students set up the initial conditions, explaining the initial conditions for the results, checking the results, and getting feedback. The finding will examine the empirical truth of students' genetic decomposition in responding to a math question or problem. As the interviewer, the author acted as a neutral observer in the interview process, allowing the subjects to express their thought processes clearly and without doubt by minimizing any contamination or influence from the interviewer.

The data analysis technique used the concept of Miles and Huberman. This model has three components: data reduction, data display, and data analysis/verification (Dunwoody et al., 2008; Goldberg et al., 2019). The data analysis phase begins after the data is collected. All collected research data is selected on what to present and use (Fajriawati & Harisman, 2020; Miles & Huberman, 1994). The data presented becomes clearer to use as a basis for drawing conclusions.

Results and Discussion

In the problem presentation stage, the students were presented with a case—the exposed surface area of a pile of cuboid-shaped boxes (as seen in Figure 1) with a side length of x, y, and z, respectively. This case is adapted from a book titled "Computational Thinking and Coding for Every Student: The Teacher's Getting-Started Guide" by (Krauss & Prottsman, 2017). The cuboid's length, height, and breadth are x, y, and z, respectively; the question given is: "Find the area of a tarpaulin needed to cover the whole exposed sides of the boxes if there are five stacks of boxes!"

Figure 1. A Case of Calculating The Area of A Few Boxes That A Tarpaulin will Cover

The result of the students' works will be interpreted using two framework references, namely, subject conceptions and the problem area experienced by the subjects. Subject conceptions are interpretations of given problems made by issues based on the subject's idea. Subject conceptions include the techniques the subjects use to solve a problem. Meanwhile, the problem area is the mistakes made by the subjects. The mistakes may be caused by random errors, consistent and patterned errors, or the impact of non-conceptual mistakes (because they forget or due to a lack of knowledge). These mistakes are often related to conceptual frameworks since new information will be stored in memory as an organized and interrelated network within a conceptual framework. One's conceptual framework is a unique and idiosyncratic idea. It is a reflection of one's explanation regarding a concept.

One's conceptual framework determines how new information will be coded. A simple framework only keeps limited information and has plenty of limitations. Meanwhile, a complex framework has adequate channels to connect new information, hence keeping more information in memory. Consequently, a subject is deemed not to have conceptual understanding if one does not take advantage of concepts that one has learned or uses related concepts but fails to implement them when solving a problem. The same applies when there is a misconception or error in describing concepts related to the given problems.

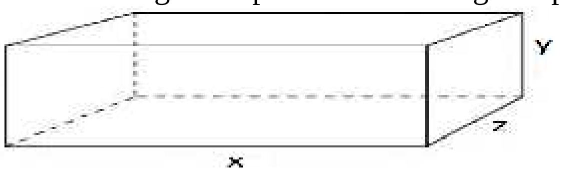

Figure 2. Initialization of The Problem Element

Based on the question given, as the first step, the students at least need to create a conceptual representation of the problem from what they know from the question: the initialization of the cuboid sides

Figures 3 and 4 show the problem-solving outcomes of Subjects A and E, respectively. The result represented by Subject A in Figure 3 shows the subject's theoretical and conceptual understanding of the given problem. It can be seen from the frequent use of polynomial formulations. Subject A tried to determine the area of the tarpaulin by decomposing the problem into its smallest part, which is one pile of the box. This is shown by how the first term is defined in the formulation, with Subject A discovering the common difference (b) by first defining the formulation for the area of the tarpaulin on each of the piles, starting from the smallest, which is described in the following process:

First pile (top box position)	: 1(xz) + 2 (xy) + 2 (yz)
Second pile	: 3(xz) + 4 (xy) + 4 (yz)
Third pile	: 5(xz) + 6 (xy) + 6 (yz)
Fourth pile	: 7(xz) + 8 (xy) + 8 (yz)
Fifth pile (bottom box position)	: 9(xz) + 10 (xy) + 10 (yz)

From this process, the formulation for the common difference is obtained: b = 2(xz) + 2 (xy) + 2 (yz). This method is considered adequate to calculate the area of the tarpaulin no matter how many piles are added.

What about subject E? It is shown in the process that Subject E demonstrated to obtain the formulation for the tarpaulin area. The result represented by Subject E in Figure 4 also shows the subject's conceptual understanding of the problem. However, it can be seen that there is a different problem-solving pattern compared to Subject A. Subject A took advantage of polynomial formulation to answer the question, while Subject E tended to use intuition in solving the problem. In mathematics, ideas that appear in the form of this intuition, of course, have a significant role in the formation of concepts (Longo & Viarouge, 2010).

Subject A tried to determine the area of the tarpaulin by decomposing the problem into its smallest parts and dividing the boxes using side representation. The total of the surface area 1 is the total of the boxes' surface sides on the front and back. The total of the surface area 2 is the total of the boxes' surface sides on the left and right. The total surface area 3 is the total of the boxes' surface sides on the top and bottom. On the other hand, Subject E manually processed the tarpaulin area calculation by drawing each side of the box in a two-dimensional form. Then the number of sides was manually calculated. Of course, this process will become more complicated if more piles are added.

The same method was also demonstrated by Subject F, as shown in Figure 5. What distinguishes Subject E's and Subject F's methods is how they represented the box in their answers. Subject F represented the box based on the point of view: front, side, and top.

How can there be a significant difference in the solution representations between Subjects A, E, and F? To answer this question, the subjects of this research were interviewed. The researcher cross-questioned students A, E, and F regarding their ideas for solving the mathematical problems presented. The following is a snippet of the discussion conducted by the researcher with subjects A, E, and F.

Researcher (R)	: Why did you answer the question using a polynomial formulation? Was it the only idea that came to your mind?
Subject A (S_A)	: At first, we also thought about using the same method as Subject F. However, it seems complicated if we add more piles. Let's say we add 100 piles of boxes; it will be a hassle to draw 100 stacks of boxes. I thought, in the end, I needed to make something general, like a polynomial case. That's why I used the polynomial formula, sir, and thankfully, the result is the same as the manual calculation.
Subject B (S_F)	: Yes, sir. At first, I planned to do the manual version. But then I thought, what if the boxes are added? Unfortunately, I did not have the foresight to do it directly as Subject A did. So, I did it part by part. The side part first, then the front, and after that, the top. I tried to move the boxes, but it looked like the result would be the same.
Subject E (S_E)	: Actually, I wanted to use the formula of an arithmetic sequence and progression and then link it with polynomials, sir, but I didn't remember the concept and operation

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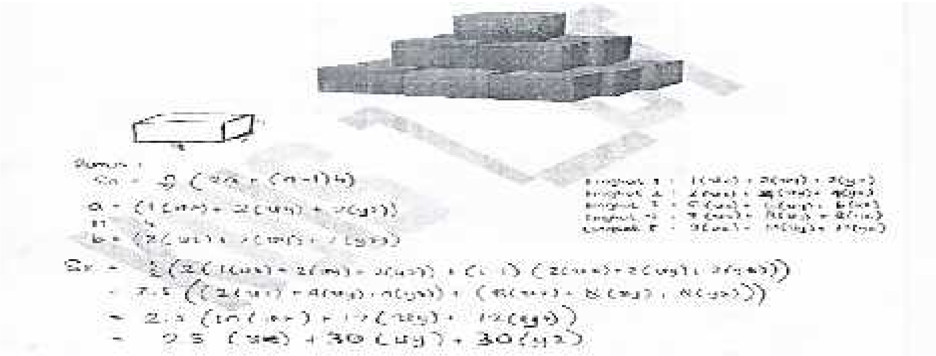

Figure 3. The Answer to The Case From Subject A

This discussion and interviews strengthen a suggestion from (Arnon et al., 2014) that one knowledge structure is constructed based on the other knowledge structure presented theoretically or based on everyone's experience in solving a particular problem. A different approach to teaching a concept will result in a different cognitive mental construction in the students. Incidentally, Subjects A and B came from a general high school, while Subjects C, D, E, and F studied at a vocational high school majoring in computer technology. Since mathematics is not considered a productive subject in vocational schools, it is possible that the student's cognitive effort in mathematics is below standard. It correlates with (Effendi, 2017), who suggests that the mathematics curriculum in vocational schools only contains a collection of materials and activities, especially in Indonesia. It is not focused on a lesson the students need, is incoherent, challenging to implement in the classroom situation vertically and horizontally, and fails to present meaningful learning. Admittedly, math lessons in vocational schools should not only be a lesson to count and calculate numbers; they should be a tool, language, or science and a means to form the students' mindset and help them adapt to society.

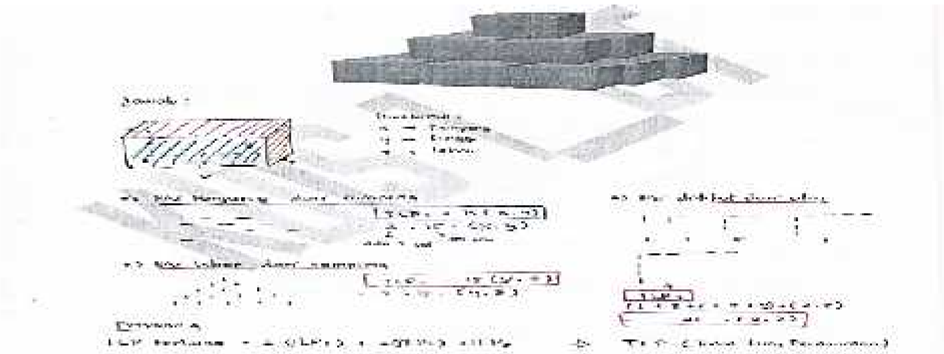

Figure 4. The Answer to The Case From Subject E

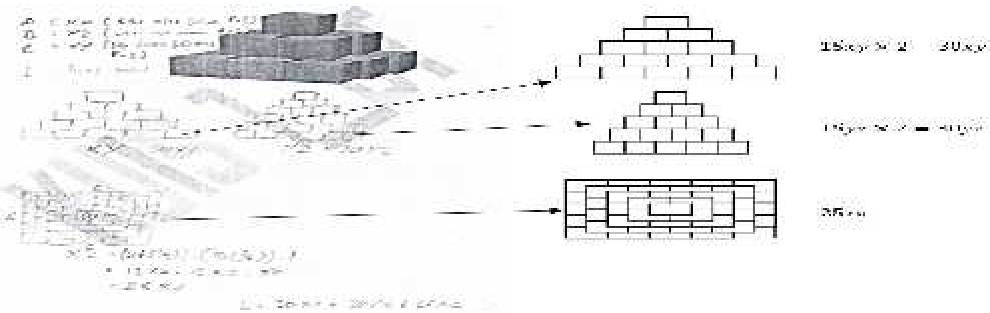

Figure 5. The Answer to The Case from Subject F

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One interesting thing from the presentation of the subjects' answers is that subjects D, E, and F's answers represent a pile of boxes in a two-dimensional drawing, like how the boxes are stacked. In contrast, subjects B and C represented their answers differently.

The interview with Subjects B and C indicates the logic of the shape transformation of piles of boxes. According to Subject B, it is possible to move the boxes. If the entire piles of boxes not in the lowest part of the stacks are shifted to one corner, the pattern of the piles will not change. Subject C states that this change does not affect the area covered by the tarpaulin. Instead, it will simplify the calculation by eliminating the parts only partially covered by the stacks above them. Meanwhile, Subjects D, E, and F only turn the three-dimensional representation of the piles into a two-dimensional representation.

The problem decomposition strategy opens opportunities for students to find the best strategy, according to them, by simplifying the problem into sub-problems. However, as we have seen, the decomposition pattern tends to be non-invariant, influenced by students' cognitive maturity in mathematics. Mathematical maturity is an intrinsic limit in implementing a problem decomposition strategy between the simplicity of representation and the optimality of the strategy used to solve the problem.

In terms of teaching characteristics, the teaching process's effectiveness is influenced by the professional skills of the teacher and the knowledge and experience of the subject matter being taught. It will eventually serve as the foundation for the student's knowledge scheme, which will be used to construct new knowledge, one of which is acquired through the problem-solving process (Papadima, 2021).

Students' knowledge is definitely built on the theoretical knowledge they learn in class and the experience they get in solving problems. Student's cognitive and mental constructions will vary based on the pedagogical approaches used to teach a concept. Students must be taught to view a problem from multiple perspectives. Consequently, a teacher needs to provide students with opportunities to explore and construct their knowledge in problem-solving, followed by a joint review. We can use the term 'pedagogical talking circles' (Barkaskas & Gladwin, 2021) to characterize patterns of interaction that may enhance students' conceptual construction and ultimately aid them in problem-solving.

When solving a problem, students usually focus on a sure way to finish it by looking at how the problem is solved in the examples they are shown. This is not entirely incorrect. The examples of problems presented and the provided problem-solving process will influence the initial perception of how to construct a problem-solving process. However, they must also be taught how to construct arguments based on the problem-solving process, which they believe can change or remain the same throughout all the stages. By teaching this, we can aid students in recognizing potential differences and similarities in the problem-solving process, enhancing their understanding of a particular concept (F. Rivera, 2013).

Obviously, this is not an easy task since students tend to focus on what they think is necessary based on what they have learned, how they have been taught, and how they have done things in the past. This phenomenon can be termed "uniformity of perception." This uniformity of perception will inevitably lead to similar settlement patterns, thereby closing the door to the possibility of a more effective and efficient solution-finding process. In the context of problem-solving, some students may need to recognize what is considered unchanged and what changes significantly (F. Rivera, 2013).

When learning math, finding and recognizing the right patterns can help bring order, make sure everyone works together, and make things more predictable. It also allows students to think beyond the information and skills they have learned. Pattern recognition is more about processes and mental habits than the content area (Clements, D. H., & Sarama, 2014).

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When students generalize patterns, they are basically coordinating their inferential perceptual and symbolic skills to build and explain structures that are logical and useful in algebra, as shown by direct formulas. Visual templates in pattern generalization activity (F. D. Rivera, 2010). It shows us that the most essential part of being a professional teacher is figuring out which parts of learning are most important to learning as a whole and using targeted instruction to improve how these parts process information.

Decomposition is the process of breaking a complicated problem down into its parts or meaningful components and then describing each part or component (Sztjan et al., 2020). Decomposition is regarded as one of the most frequently utilized heuristic problem-solving techniques. Using decomposition, seemingly insurmountably complex problems become much more manageable. The decomposition process can be applied iteratively to each sub-problem down to the most fundamental sub-problem, allowing us to analyze various facets of potential solutions, guide our thinking, and arrive at the optimal solution.

Because decomposition can be used in many ways, finding an appealing and practical general framework is challenging. We can determine the efficacy of this decomposition procedure by examining factors such as the observed object's dispersion or its compatibility with what we already know about a particular field of study or subject (Tuncer et al., 2008). Decomposition can also be understood as a hypothetical model that describes the mental structures and mechanisms students may need to construct to master a specific concept (Arnon et al., 2014). (Arnon et al., 2014) referred to this phenomenon as "genetic decomposition." Why is this model referred to as hypothetical? Because the decomposition process tends to be influenced by the conceptual, theoretical, and practical learning experiences that students have acquired, it is only natural that decomposition can also be used to explain the extent to which students' knowledge has grown and developed, as well as to demonstrate individual differences in the development of knowledge construction.

This decomposition also differentiates the solution-finding process from the presented "cardboard pile" problem. The pile is then split on each side—the front, the left, right, and the top—to process a part of the solution only found through intuitive decomposition. Using elementary arithmetic, formula solutions are obtained. Others attempt to incorporate a creative process into the solution determination, such as shifting the pile formation. Others attempt to assign a discernible pattern to the decomposition process. This recognition of patterns leads students to formulate solutions by associating them with the concept of sequences. Using the term proposed by (Sztjan et al., 2020), which is actually employed in the decomposition of teaching activities to obtain the optimal process, students who tend to seek the first solution demonstrate the existence of decomposition at the domain level. The domain level focuses more on the decomposition procedure, which involves only known situations and data. Students' propensity to seek the second solution indicates the presence of decomposition at the organizational level. The organizational level is more focused on implementing specific strategies that facilitate the problem decomposition procedure and make it simpler to represent solutions. In the meantime, students with the last trend demonstrate the existence of a technical-level decomposition process. The technical level is more focused on implementing specific strategies acquired through literacy and previous processes.

Conclusion

Decomposition skill is influenced by how basic mathematics skills are taught and absorbed by students at a lower level of education. In other words, teaching characteristics influence students' mindsets regarding understanding and solving a specific math problem. At this point, the teaching characteristics will determine the students' maturity in interacting with mathematical concepts. So, the problem decomposition skills, mathematical maturity, and their relation to solving mathematical problems in computer science learning classes influenced one another. Concerning problem decomposition, the ability to decompose a problem is essential for the students to learn because problem decomposition seems like a guarantee to find a solution for every problem. Involving decomposition skills in problem-solving can help reveal the solutions to the problems or at least make the problems look easier in smaller parts (as a part of a series of steps to complete the problem) rather than looking at it as a whole. This study also reveals that the conception construction benefits students in understanding how assimilation and accommodation occur in human cognitive structure.

Recommendation

There is a need for further research on the gap in the literature on learning mathematics in public and vocational schools. Mathematics learning in vocational schools has not been thoroughly studied, especially in terms of the position of mathematics in the structure of the vocational education curriculum and the contribution of learning mathematics to the formation of mindsets that can support productive vocational subjects. Adopting decomposition skills in mathematics can impact computational strategies related to the role of the computer as a tool for solving problems. So that learning mathematics at the vocational level in the future will not only count and calculate numbers but, at the same time, as a suggestion to shape students' mindsets and help them adapt to society.

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