



## Optimization of Rice (*Oryza sativa*) Crop in Simalungun District using Quadratic Programming Wolfe Model and Exterior Penalty Function Method

\*Wulandari, Riri Syafitri Lubis

Faculty of Science and Technology, Mathematics Study Program, North Sumatra State Islamic University

\*Corresponding Author e-mail : [bpurnama537@gmail.com](mailto:bpurnama537@gmail.com)

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### Abstract

Optimization is a way of optimizing the objective function while still paying attention to existing constraints. The aim of this research is to form a mathematical model to optimize the average production of lowland and field rice in Simalungun Regency and to complete the model using *Wolfe model quadratic programming* and the exterior penalty function method. The mathematical model in this research is a nonlinear model created using the least squares method. Quadratic programming solves nonlinear problems by turning them into linear problems using the *Kuhn Tucker condition* and linear problems are solved using the *Wolfe model*. Meanwhile, the exterior penalty function method solves limited (constrained) nonlinear problems into unlimited (unconstrained) nonlinear problems. Based on the calculations of the two methods, optimal results were obtained for both the *Wolfe* quadratic programming model and the exterior penalty function method, where the average production of wetland and field rice in Simalungun Regency was 203.1925 tonnes with a wetland rice harvest area of 90.077 hectares and a field rice harvest area of 13.092. hectare. Optimization testing using this algorithm can be used as a basis for related parties such as the community and government in developing productivity. The implication of this research is an increase in harvested area and average rice production, in this case maintaining productivity stability.

**Keywords:** Optimization, Least Squares Method, Quadratic Programming, Exterior Penalty Function Method

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## INTRODUCTION

Indonesia is a country where the majority of its population depends on rice as its staple food. Thus, it cannot be denied that the public's demand for rice will be very large, but in fact the existing centers in Indonesia are still unable to meet the public's demand for rice. Indonesia, which is supposed to be an agricultural country, apparently has to import rice from various countries, this is certainly an anomaly that must be addressed immediately. (Indainanto, Nasution, Fauzan, Ardian, & Dalimunthe, 2022)

Indonesia's powerlessness in meeting rice needs in Indonesia is caused by the lack of understanding of production limits between plantations at the farmer level. Plant development is a movement passed down from ancestors, but along with increasing innovation and information, farmers must be able to train good and attractive planting systems to obtain maximum results, so that domestic food needs can be met and security is guaranteed. (Ruvananda & Taufik, 2022).

As a food crop, around 90% of the total Indonesian population consumes rice as a daily staple food. According to data from the Simalungun Regency Central Statistics Agency (BPS) for 2016-2022, rice food production varies from year to year, even though demand for food is

increasing every year. Therefore, the government must look for alternatives to meet these needs. One way that can be done is to maximize agricultural yields. So research was carried out to optimize agricultural results.

Simalungun Regency is a district located in the province of North Sumatra, Indonesia. Simalungun Regency's greatest potential lies largely in its agricultural production. During 2016-2019 Simalungun Regency produced rice in the quantities of 634,828.3 tonnes, 447,135 tonnes, 472,440 tonnes and 336,322 tonnes respectively. This causes instability in average production each year (BPS, 2021).

However, it turns out that rice productivity fluctuates every year. This danger is caused by several aspects, including: general rainfall, large plantings, production, large harvests, and generally strong windy days. This can cause chaos if something happens when production power experiences a significant reduction. Therefore, productivity needs to be increased so that it can be properly assessed what the ideal future productivity will be.

Optimization is an important problem that is often experienced in everyday life, so there are many problems in everyday life that require an improvement approach in solving them. This problem is divided into two types, namely optimization problems without constraints and optimization problems with constraints. If the ability or natural requirement is nonlinear, it is called nonlinear programming, whereas if the ability is linear, it is called direct programming (Erlina, Syaripuddin, & Tisna Amijaya, 2022).

There are two types of optimization approaches for constrained nonlinear problems: direct and indirect approaches. One technique that is direct is quadratic programming, while one that is indirect is the exterior penalty function method. Nonlinear programming problems are of great interest to researchers because of their application in many fields such as financial planning, production and companies.

According to (Wright, Stephen J: 2015) programming is based on formal procedures for solving mathematical problems. This use began in the 1940s and is not specifically associated with current ideas about *computer programming*. Some experts prefer to use the term optimization to avoid confusion.

method was chosen because after being converted into linear form, the simplex method can be used to complete quadratic programming. Quadratic programming in this case will later be transformed into a linear problem by referring to the *Kuhn Tucker conditions* which are then solved using the *Wolfe simplex*. In this method there is *complementary slackness*, this is what differentiates the *Wolfe method* from ordinary simplex.

According to (Rao in Insani: 2017) the penalty method was chosen because this method can determine the constraint problem more broadly which solves the constrained nonlinear optimization problem without constraints by adding a penalty function and parameters to the objective function. Apart from that, this method is used in solving problems of equality and inequality constraints. Meanwhile, interior penalties only solve problems with inequality constraints.

Insani (2017) has conducted research that expands food crops in the city of Magelang by involving quadratic programming and the exterior penalty method. Larita (2018) optimizes rice production using the *Wolfe method of quadratic programming*. RH Putri, et al (2021) analyzed the optimal stock portfolio using the *Wolfe method*. Furthermore, Fitri & Helma (2023) optimizes average corn production using the *Wolfe method of quadratic programming*.

The aim of this research is to form a mathematical model to optimize the average production of wetland rice and field rice in Simalungun Regency using the *Wolfe model* of quadratic programming and the exterior penalty function method. The benefit of this research is to produce optimal harvest area for paddy fields and fields in Simalungun Regency so that it can later be used as a basis for increasing the productivity of rice plants and used as reference material in subsequent linear programming optimization studies.

## METHOD

### Optimization

Optimization is a standard method for determining the most effective solution when drawing conclusions from a case. Solving optimization problems aims to obtain maximum or minimum points from the optimized function (Haida, 2021).

Several optimization methods include: Linear Programming Optimization is a technique for achieving the best results (such as the largest profit or minimum payment) in a numerical structure where all fundamental requirements are introduced in a linear relationship .

Stochastic optimization methods are streamlining systems that create and use irregular factors. For stochastic problems, irregular factors appear in the details of the progress problem itself, including arbitrary reasonable qualities or arbitrary requirements, for example. Some stochastic advancement strategies use arbitrary suppression to overcome stochastic problems, combining the two is called stochastic improvement. Stochastic improvement techniques encapsulate deterministic strategies for issues that are deterministic in nature.

Optimization using the Calculus of Variations is a subfield of mathematical analysis that deals with maximization and minimization problems of functions that describe the combination of abilities in natural numbers. Utilitarianism is often expressed as an indispensable thing that connects capabilities and their subsidiaries. What is really of interest are the extreme points or extreme points of the function, where the function produces a maximum, minimum, or stationary sum of functions and the change in function is usually zero.

Genetic Algorithm Optimization is a heuristic attempt that mimics normal regulatory approaches. This trademark heuristic is consistently used to produce valuable answers to repair and search problems. Hereditary calculations are remembered for a larger classification of moderate calculations, which provide answers to improvement problems using techniques motivated by normal progress, such as narrowing, relocation, setting and crossing points. (M & Subanar, 2017).

### Nonlinear Programming

Problems in everyday life cannot always be solved with linear programming, so nonlinear programming emerged. The objective function in nonlinear programming must be nonlinear, while the constraint functions can be linear or nonlinear.

The general form of a linear programming problem is to find the value of a decision variable  $x_1, x_2, \dots, x_n$  in order to maximize or minimize it

$$\text{Objective function } f(x_1, x_2, \dots, x_n), \quad (2.1)$$

With constraints

$$g_1(x_1, x_2, \dots, x_n) (\leq, =, \text{ atau } \geq) b_1 \quad (2.2a)$$

$$g_2(x_1, x_2, \dots, x_n) (\leq, =, \text{ atau } \geq) b_2 \quad (2.2b)$$

⋮

$$g_m(x_1, x_2, \dots, x_n) (\leq, =, \text{ atau } \geq) b_m \quad (2.2c)$$

$$x_1, x_2, \dots, x_n \geq 0$$

With  $f$  a nonlinear function and  $g$  a linear or nonlinear function. (WinsTon in Humans, 2017: 19)

Nonlinear programming is divided into three parts. First, Nonlinear Programming Without Constraints is an improvement that does not require nonlinear goals. A type of nonlinear programming without barriers to determining numbers  $(x_1, x_2, \dots, x_n)$  with

Objective function : maximum / minimum

$$f(x_1, x_2, \dots, x_n)$$

Optimal conditions where nonlinear programming problems can be solved without constraints. The main condition is the necessary condition for optimality which is used to find the optimal points  $x^*$  in the analytical approach. The following is **definition 2.1** from the necessary conditions for optimality:

*If the solution  $x = x^*$  is the optimal point of  $f(x)$  then:*

$$\frac{\partial f}{\partial x_j} = 0, \text{ wheres } x = x^*, \text{ and } j = 1, 2, \dots, n$$

The second condition is the sufficient optimality condition where this condition is used to determine whether the maximum point obtained from the sufficient optimality condition is the minimum point or the maximum point used by the sufficient optimality condition. The following is **definition 2.2** from the conditions of sufficient optimality

If  $\frac{\partial f}{\partial x_j} = 0$ , dan  $H(x^*)$  definit positif maka  $x^*$  titik minimum

If  $\frac{\partial f}{\partial x_j} = 0$ , dan  $H(x^*)$  definit negatif maka  $x^*$  titik maksimum

Second, nonlinear programming with linear constraints where optimization is carried out using linear functions as constraints and nonlinear functions as goals in nonlinear programming with linear constraints. To determine the value  $x = x_1, x_2, \dots, x_n$  in general form is:

$$\begin{array}{ll} \text{Maximum/minimum} & : (x_1, x_2, \dots, x_n) \\ \text{Constraints} & : g_m(x) (\leq, =, \geq) 0, \text{ for } m = 1, 2, \dots, n \end{array}$$

Third, nonlinear programming with nonlinear constraints is a downsizing problem with indirect goals and limitations. There are two types of nonlinear programming with nonlinear constraints:

First, For  $x_1, x_2, \dots, x_n$  general form of nonlinear programming with similarity constraints: Objective function

$$\text{Maximum/minimum: } f(x_1, x_2, \dots, x_n)$$

$$\text{Constraint function : } g_m(x) = 0$$

Where m indicates the number of constraints and n indicates the number of variables with  $m \leq n$ . Then secondly, the general form of nonlinear programming with inequality constraints:

$$\text{Maximum/minimum: } f(x_1, x_2, \dots, x_n)$$

With the constraints:  $g_m(x) (\leq, =, \geq) 0$ , for  $m = 1, 2, \dots, n, x \geq 0$

### Karush Kuhn-Tucker Terms

Karush Kuhn Tucker (KKT) characterized refinement techniques in 1951, where the strategy can be used to find the ultimate goal in something that has both linear and nonlinear limitations. The questions involving quadratic programming are based on KKT requirements .

#### Theorem 2.1

Suppose  $f(x)$  and  $g_i(x)$  is a maximization problem. If  $x^*(x_1^*, x_2^*, \dots, x_n^*)$  is the optimal solution for  $f(x)$  and  $g_i(x)$ , then  $x^*(x_1^*, x_2^*, \dots, x_n^*)$  must satisfy (1) with constraints of the form  $g_i(x) \geq b_i$ , and there must be a multiplier  $\lambda_1, \lambda_2, \dots, \lambda_m$  and slack variable  $s_1, s_2, \dots, s_n$  that satisfies:

$$\frac{\partial f(x^*)}{\partial x_j} - \sum_{i=1}^m \lambda_i \frac{\partial g_i(x^*)}{\partial x_j} + s_j = 0, \quad j = 1, 2, \dots, n \tag{2.3a}$$

$$\lambda_i [b_i - g_i(x)] = 0 \quad (i=1, 2, \dots, m) \tag{2.3b}$$

$$\left[ \frac{\partial f(x^*)}{\partial x_j} - \sum_{i=1}^m \lambda_i \frac{\partial g_i(x^*)}{\partial x_j} \right] x^*_j = 0 \quad (j=1, 2, \dots, n) \tag{2.3c}$$

$$\lambda_i \geq 0 \quad (i=1, 2, \dots, m)$$

$$s_j \geq 0 \quad (j=1, 2, \dots, n)$$

According to (Hiller in Insani, 2017: 24) For forced matters, the Kuhn Tucker condition is a condition that requires optimization, and will be an adequate condition if the goal is curved and the limits are increased .

### Linear Programming

Linear programming is a technique for determining the highest value of a problem directly. The highest (most extreme or smallest) number is obtained from the numbers in the combination of the straight problems faced. There is a linear meaning, which is usually called a natural meaning, in the linear case. As far as possible and the conditions in the direct problem are a straight arrangement of imbalances

Equations and inequalities in linear programming, by definition must be linear. Linear equations have a general form

$$a_0 + a_1x_1 + a_2x_2 + \dots + a_nx_n = 0 \quad (2.4)$$

In general, they are called equation coefficients, or also called parameters. Fixed-valued coefficients based on the basic nature of the problem being solved,  $x$  are called equation variables and can take a range of values within the limits determined by the constraints.

In general, the linear programming problem is formulated as follows

Maximize/minimize

$$f = CX \quad (2.5)$$

With constraints

$$AX (\leq, =, \geq) B, X \geq 0 \quad (2.6)$$

Where ;

$$C = [c_1, c_2 \dots c_n] \quad (2.7)$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad (2.8)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad (2.9)$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

### Quadratic Programming

Nonlinear programming with quadratic objectives and linear constraints is known as quadratic programming. The purpose or constraints in connecting the square of an elastic  $x_j^2$  or the product of two elastics can be determined by comparing quadratic programming problems with linear programming problems. A conventional quadratic programming problem can be formed as follows:

$$\text{Minimizing } f(x) = CX + \frac{1}{2}x Dx + d \quad (2.10a)$$

$$\text{Subject to } a_i^T x \leq b_i \quad (2.10b)$$

$$a_i^T x = b_i \quad (2.10c)$$

$$x \geq 0$$

Matrix  $X$  is a one-column matrix of the variables to be searched.  $C$  is a single row matrix of cost coefficients ( $c_j$ ), matrix  $A$  is the coefficient matrix of the constraint equation and  $B$  is the column matrix of the right side of the constraint equation.

### Simplex Method

The simplex method was proposed for the first time in 1947 by GB Dantzig [Hua90]. A simplex strategy is a logical system that starts from a basic solution which can be continued to other basic solutions and tried more than once (iterative) in order to achieve a maximum solution. (Herjanto in Putri, 2021:22)

The steps in solving the simplex method include : First, change the form of the constraint function into an equation (canonical form) by adding a *slack variable*. Forms of constraints in the simplex method; to ( $\leq$ ) be converted into an equation form by adding a *slack variable*, to ( $\geq$ ) be converted into an equation form by reducing the *surplus variable* and then adding an artificial variable and to ( $=$ ) be solved by adding an artificial variable. *Second*, arrange the equations into a simplex Table 1.

**Table 1.** Simplex table

	$c_i$	$c_1$	$c_2$	...	$c_n$		
$c^*_j$	$v^*_j/v_i$	$v_1$	$v_2$	...	$v_n$	$b_j$	$r_j$
$c^*_1$	$v^*_1$	$a_{11}$	$a_{12}$	...	$a_{1n}$	$b_1$	$r_1$
$c^*_2$	$v^*_2$	$a_{21}$	$a_{22}$	...	$a_{2n}$	$b_2$	$r_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
$c^*_m$	$v^*_m$	$a_{m1}$	$a_{m2}$	...	$a_{2n}$	$b_m$	$r_m$
	$z_i$	$z_1$	$z_2$	...	$z_n$		
	$z_i - c_i$	$z_1 - c_1$	$z_2 - c_2$	...	$z_n - c_n$		

Information :

$v_i$ = decision variable

$v^*_j$ = base variable

$c_i$ = coefficient of the decision variable( $x_i$ )

$c^*_j$ = coefficient of the base variable( $x^*_i$ )

$a_{ij}$ = constrained coefficient

$b_j$ = quantity

$z_i$  = total number of row products  $c^*_j$ with the base variable

$z_i - c_i$ = the difference between the two values

$r_j$ = ratio (quotient of the value  $b_i$  with base variables )

*Third*, complete the simplex table . The following are the steps in completing a simplex table : Checking optimal values , determining key variables, and compiling a new simplex table.

In the maximum case, the simplex table is declared optimal if  $z_i - c_i \geq 0$ for all values of  $i$  . Meanwhile, in the minimum case, the simplex table is declared optimal if  $z_i - c_i \leq 0$ . If the table is not optimal, then iterative improvements are carried out. To determine the key variable, first determine the key column and key row. Determining the key column is by looking at the value  $z_i - c_i$ , while determining the key row is by looking at the ratio.

In the maximum case, determining the key column is by selecting the  $x_{ij}$  value that has  $z_i - c_i < 0$ the smallest, while determining the key row is by selecting the smallest positive ratio value. In the minimum case, determining the key column is by selecting the  $x_{ij}$  value that has  $z_i - c_i > 0$ the largest value, while determining the key row is by selecting the smallest positive ratio value. The intersection of the values between the key column and the key row is then used as the key variable value.

To construct a new simplex table, the first step is to find the coefficients of the base elements of the previous simplex table. The basic element coefficients are the values of key variables that have been previously determined . So the new private row coefficient can be found using the formula  $\frac{a_{rj}}{a_{rk}}$ , where  $a_{rj}$ is the value of the base variable, and  $a_{rk}$ is the value of the key variable. Meanwhile, to calculate other new row values, use the formula  $a_{ij} - \frac{a_{rj}}{a_{rk}} a_{ik}$ . Table iteration is carried out until it reaches optimal conditions.

**Two Phase Simplex Method**

This method is a technique that can overcome linear programming problems by utilizing table iterations which are completed in two phases where the initial base variables consist of artificial variables. The basic stage is to eliminate artificial variables by creating an artificial objective function , namely the size of the artificial variable which is then limited using a simplex table. In the maximum problem , the objective function coefficient is given a negative value, while in the minimum problem the objective coefficient is given a positive value.

According to the rules of the two-phase simplex procedure , The first phase will be completed if the objective function (artificial) has a maximum number. Next, the strategy can

be continued in the second phase. In the next stage, the initial table is used as an optimization of the original objective function derived from the final table in the first phase.

The steps of the two-phase simplex method include ; In the initial phase, create an artificial objective function. Then, transform the constraint function in canonical form. Create and complete a simplex framework and then carry out simplex solutions until an optimal condition is obtained. In phase two, the objective values  $c_i$  and the values  $c_j^*$  will follow the coefficients of the original objective function. Then, complete the simplex table until optimal conditions are obtained .(Putri, Sudarwanto, & Wiraningsih, 2022)

### Wolfe Model

*Wolfe* is a system used to solve quadratic programming problems. This strategy requires *the Karush Kuhn Tucker* (KKT) state as a condition for creating a new linear goal, then is limited by involving the initial phase in a two-phase simplex technique . In mathematical optimization, *Wolfe duality* is also known as *Philip Wolfe* , which is a type of dual problem where the objective function and constraints are differentiable functions.

*Wolfe* method process will begin by adding artificial variables to the equation results obtained from the KKT conditions. Then the artificial variable will be minimized as a new objective function;

$$w = a_1 + a_2 + \dots + a_n \quad (2.11)$$

Meanwhile, the obstacle is the collection of results obtained from meetings within the KKT environment. After that the case can be entered into the simplex table.

To guarantee that the final (versatile array created by  $a_i = 0$ ) fulfill the circumstances *complementary slackness* , *Wolfe's technique* has changes in the choice of simplex variables included as bases , in particular:  $s_i$  of the KKT state and the decision variable  $x_i$  cannot be the basis variable simultaneously . Good advantages variable  $e_i$  nor  $s_i$  from the  $i$ -th constraint and  $\lambda_i$  from the KKT conditions, both cannot be basic variables.

Conditions above correspond to *the complementary slackness* of quadratic programming. Furthermore , if the simplex is done with the usual strategy without taking advantage of the basic conditions above, then in the optimal table there will be *complementary slack* that is not fulfilled.

### Exterior Penalty Function Method

Exterior penalty function method is a method in which a constrained nonlinear optimization problem is transformed into an unconstrained problem by adding a penalty function and parameters in the objective function.

The exterior penalty function is an objective function that contains a penalty function :

$$\alpha(x) = \sum_{i=1}^m [\max\{0, g_i(x)\}]^p + \sum_{i=1}^l |h_i(x)|^p \quad (2.12)$$

So the general form of the exterior penalty function problem is:

Minimize

$$z = f(x) + \mu_k \sum_{i=1}^m [\max\{0, g_i(x)\}]^p + \mu_k \sum_{i=1}^l |h_i(x)|^p \quad (2.13)$$

Information:

$f(x)$  is the objective function of the constrained problem

$g_i(x)$  is the inequality constraint function

$h_i(x)$  is the constraint function of the equation

$p$  is a positive integer

$\mu_k$  is a penalty parameter

The steps in solving the exterior penalty function method are to form the objective function of an unconstrained optimization problem according to the general form of the equation. Then, determine the solution to the problem of minimizing  $z$  , namely,  $x^*$  . Next, determine whether the number  $x_1$  dan  $x_2$  is the minimum or maximum number based on the conditions for optimizing a nonlinear problem without constraints . Here's how to determine it:

$$\text{If } \frac{\partial f}{\partial x_j} = 0 \text{ and } H(x^*) \text{ positive definite then } x^* \text{ is the minimum point}$$

$$\text{If } \frac{\partial f}{\partial x_j} = 0 \text{ and } H(x^*) \text{ negative definite then } x^* \text{ is the maximum point}$$

## RESULTS AND DISCUSSION

### Formation of Objective Functions using the Least Squares Method

The problem studied in this final assignment is a nonlinear programming problem, so the intention to be made is nonlinear ability in the form of quadratic ability which is the highest level ability and its factor is 2. Quadratic ability has one peak point number (the most extreme and the smallest). Least squares methodology is used as a standard planning technique that limits the number of remaining squares.

Models completed using this method include the following.

$$y = x\beta + \varepsilon \quad (3.1a)$$

Where  $\beta$  are the parameters and  $\varepsilon$  the remainder (error). According to Setijo Bismo (in Insani, 2017: 46), the regular form of a quadratic function or parabola can be written as follows:

$$y = \beta_0 x^2 + \beta_1 x + \beta_2 \quad (3.1b)$$

From collecting 3.1a and 3.1b, a quadratic structure is obtained which will be solved using the least squares technique, more specifically:

$$y = \beta_0 x^2 + \beta_1 x + \beta_2 + \varepsilon \quad (3.1c)$$

In this case, the least squares method is used to find constant values  $\beta_0$ ,  $\beta_1$ , dan  $\beta_2$  based on the given data set.

### Nonlinear Problem Solving

#### *Quadratic Programming with the Wolfe Model*

Quadratic programming handles nonlinear programming problems by turning them into direct programming problems that contain *Kuhn Tucker's guidelines*. The current prerequisites obtained will be to find the ideal arrangement using *Wolfe's simplex structure*.

The steps include the following: Forming *Karush Kuhn Tucker conditions*, identifying *complementary slackness* according to the properties of quadratic programming, adding variables  $a_i$  in each KKT condition, creating a new linear goal to limit the number of variables  $a_i$ , and completing an iterative strategy using the *Wolfe structure*.

To guarantee that the final solution (variable  $a_i = 0$ ) satisfies the *complementary slackness condition*, *Wolfe's model* has changes to the choice of simplex variables included as a basis, specifically:

“  $s_i$  from *Karush Kuhn Tucker conditions* and decision variables  $x_i$  cannot be basis variables simultaneously.

The surplus variable  $e_i$  or slack variable  $s_i$  from the  $i$ th constraint and  $\lambda_i$  from the *Karush Kuhn Tucker condition* cannot both be base variables.

Substitute the results from the optimal line into the initial objective function to obtain optimal results.

#### *Exterior Penalty Function Method*

The exterior penalty function method is basically a technique that transforms a forced nonlinear problem into an unconstrained problem, so that the solution is sought mathematically. A constrained problem will be transformed into an unconstrained problem by entering a penalty function and its parameters

The steps for solving the exterior penalty method include: Forming an objective function using the exterior penalty method

$$z = f(x) + \mu_k \alpha(x),$$

with :

$f(x)$  is the objective function of a constrained problem

$\mu_k$  is a penalty parameter

Penalty function

$$\alpha(x) = \sum_{i=1}^m [\max\{0, g_i(x)\}]^p + \sum_{i=1}^l |h_i(x)|^p$$

$g_i(x)$  is the inequality constraint function

$h_i(x)$  is the constraint function of the equation

$p$  is a positive integer

Then determine the solution to problem z, namely  $x^*$  and check whether the optimal value obtained is the maximum or minimum value based on the conditions for sufficient optimality.

### Application of the Model to Average Rice Crop Production in Simalungun Regency

The following is the formation of a mathematical model for optimizing the average rice crop in Simalungun district.

#### Model Formation

The data used is data obtained from the Central Statistics Agency (BPS) for the Simalungun region. The data studied consisted of data on land area, harvest area, and average rice production in 2016-2022 .

**Table 2.** Data on Land Area, Harvested Area, and Average Production of Lowland Rice and Field Rice in 2016-2022

Year	Rice Fields			Rice Fields		
	LL (Ha)	L.P (Ha)	R PP (Tons)	LL(Ha)	L.P (Ha)	R PP (Tons)
2016	35250	102	61.94	-	10	34.26
2017	33150	71	62.18	-	17	34.17
2018	31253	77	60.66	-	15	44.99
2019	31273	62	53.73	49838	9	33.75
2020	-	-	-	-	-	-
2021	-	-	57.91	48922	8	39.21
2022	30749	61	57.75	48922	12	40.16

Information :

LL : Land area

LP : Harvest Area

RRP : Average Production

Based on Table 2, the average rice production every year always changes, so the mathematical model that will be formed is a nonlinear function .

#### Forming the objective function

For the reasons behind this, production is generally characterized by collection per hectare, where the weight is determined by tonnes. After all, the size of the harvest is the number of plants collected after they are old, including those that are neglected to collect.

Based on the data in table 3.1, this will be done using a least squares strategy determined using the Matlab application. There are also steps, including those that accompany entering size data for collecting and creating transitions into the Matlab application or can be written using instructions in the *command window*. Then type quadratic programming *syntax* to get the objective function parameters. The next step is to call *the script* in *the command window* , then press enter and the results will appear in *the command window* as below

```

Command Window
>> m =
      6

n =
      2

A =
    211275956    2567942    32660
    2567942     32660    32660
    32660         434         2

Y =
    1.0e+06 *
    1.9544
    0.0258
    0.0004

beta =
    -0.0078
     1.4052
    -0.0000

```

**Figure 2.** Display of output from paddy fields

```

Command Window
>> m =
      6

n =
      2

A =
    175539    12257    903
    12257     903    903
     903         71     2

Y =
    1.0e+04 *
    3.4450
    0.2698
    0.0227

beta =
    -0.2370
     6.2055
    -0.0005

```

**Figure 3.** Display of output from rice fields

The results from Figures 2 and 3 were obtained with the final aim of this issue to amplify the development of lowland rice and field rice, so that the common goal is to carry out augmentation.

$$f(x) = f(x_1) + f(x_2)$$

$$\Leftrightarrow f(x) = [-0,0078x_1^2 + 1,4052x_1 - 0,0000] + [-0,2370x_2^2 + 6,2055x_2 - 0,0005]$$

$$\Leftrightarrow f(x) = -0,0078x_1^2 - 0,2370x_2^2 + 1,4052x_1 + 6,2055x_2 - 0,0005 \tag{3.2}$$

Next, we will first investigate whether equation 3.2 is valid or not, namely by ensuring that the determinant  $\neq 0$  and *conditional number* are below 67108864, then the beta coefficient value in the objective function is declared the best.

**Table 3.** Results of analysis of matrix determinants and *conditional numbers* for lowland rice and lowland rice.

	Rice Fields	Rice Fields
Determinant of matrix A	-2.5338	-1.1938
<i>Conditional numbers</i>	5.6906	2,1874

Based on the table above, we get the determinant results for both rice plants  $\neq 0$ , so that the system of linear equations has a single solution. While *the Conditional number* for both rice plants is  $< 67108864$  or below the maximum limit, then equation 3.2 is declared the best.

**Forming a Constraint Function**

In this situation, the problem is that the harvest area cannot be more than the largest land area. Due to table 3.1, the significance of the obstacles to this problem are:

$$g_1(x) = x_1 \leq 35250 \tag{3.3a}$$

$$g_2(x) = x_2 \leq 49838 \tag{3.3b}$$

$$x_1, x_2 \geq 0 \tag{3.3c}$$

So the numerical structure of the overall production of the central factory in Simalungun Regency is a nonlinear structure with objective 3.2 and condition 3.3.

**Completion with Wolfe Model Quadratic Programming**

Before completing quadratic programming using the *Wolfe structure*, intersection points 3.2 and 3.3 will be distinguished in typical types of quadratic programming.

Considering equation 2.5, up to equation 3.2 can be arranged as follows:

$$C = [1,4052 \quad 6,2055 \quad -0,0005]$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$D = \begin{bmatrix} -0,0156 & 0 \\ 0 & -0,474 \end{bmatrix}$$

$$d = -0,0005$$

So it is obtained

$$f(x) = [1,4052 \quad 6,2055 \quad -0,0005] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2} [x_1 \quad x_2] \begin{bmatrix} -0,0156 & 0 \\ 0 & -0,474 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 0,0005$$

The problem is

$$g_1(x) = x_1 \leq 35250$$

$$g_2(x) = x_2 \leq 49838$$

$$x_1, x_2 \geq 0$$

Equations (3.2) and (3.3) are in accordance with the general form of quadratic programming problems with equation (3.2) being a concave function, while equation (3.3) is a convex function so that the *Kuhn Tucker condition* can be used as a necessary and sufficient condition for optimality (Hillier in Insani, 2017 : 47). Therefore, equations 3.2 and 3.3 can be solved by *Wolfe model quadratic programming*.

*First*, form the *Kuhn Tucker Condition*

Based on theorem 2.1, the *Kuhn Tucker condition is obtained*, namely:

$$-0,0156x_1 + 1,4052 - \lambda_1 + s_1 = 0$$

$$-0474x_2 + 6,2055 - \lambda_2 + s_2 = 0$$

In the equation above, it can also be written as:

$$0,0156x_1 + 1,4052 - \lambda_1 - s_1 = 0 \tag{3.4a}$$

$$0474x_2 + 6,2055 - \lambda_2 - s_2 = 0 \tag{3.4b}$$

$$\lambda_1[35250 - x_1] = 0 \tag{3.5a}$$

$$\lambda_2[49838 - x_2] = 0 \tag{3.5b}$$

$$(-0,0156x_1 + 1,4052 - \lambda_1)x_1 = 0 \tag{3.6a}$$

$$(-0,474x_2 + 6,2055 - \lambda_2)x_2 = 0 \tag{3.6b}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{3.7}$$

$$s_1, s_2 \geq 0 \tag{3.8}$$

Based on equation 3.5, it is obtained:

$$x_1 - 35250 \leq 0 \tag{3.9a}$$

$$x_2 - 49838 \leq 0 \tag{3.9b}$$

The form of equation 3.9 can be used as a canonical form so that it becomes

$$x_1 + s'_1 = 35250 \tag{3.10a}$$

$$x_2 + s'_2 = 49838 \tag{3.10b}$$

After identifying the *Kuhn Tucker condition*, the KKT for equation 3.2 is

$$0,0156x_1 + 1,4052 - \lambda_1 - s_1 = 0 \tag{3.4a}$$

$$0,474x_2 + 6,2055 - \lambda_2 - s_2 = 0 \tag{3.4b}$$

$$x_1 + s'_1 = 35250 \tag{3.10a}$$

$$x_2 + s'_2 = 49838 \tag{3.10b}$$

*Second*, identify *complementary slackness*

Based on equations 3.5 and 3.10, equation 3.3 and equation 3.4 and the *complementary slackness characteristic* for equation 3.2 are

$$\lambda_1 s'_1 = 0 \qquad s_1 x_1 = 0$$

$$\lambda_2 s'_2 = 0 \qquad s_2 x_2 = 0$$

*Third*, adding artificial variables  $a_i$  to equation 3.4 is intended for every *Kuhn Tucker condition* that does not have a base variable. so that the variables created by  $a_i$  are added and the shape becomes

$$0,0156x_1 - \lambda_1 - s_1 + a_1 = 1,4052 \tag{3.11a}$$

$$0,474x_2 - \lambda_2 - s_2 + a_2 = 6,2055 \tag{3.11b}$$

*Fourth*, determine a new linear objective function

Minimize

$$w = a_1 + a_2 \tag{3.12}$$

With constraints

$$0,0156x_1 - \lambda_1 - s_1 + a_1 = 1,4052 \tag{3.11a}$$

$$0,474x_2 - \lambda_2 - s_2 + a_2 = 6,2055 \tag{3.11b}$$

$$x_1 + s'_1 = 35250 \tag{3.10a}$$

$$x_2 + s'_2 = 49838 \tag{3.10b}$$

*Fifth*, carry out simplex iterations with the *Wolfe model*.

After obtaining the new objective function and constraints, namely equations 3.10 to 3.12, a simplex table is created and the calculations are carried out.

**Table 4.** *Wolfe model initial simplex table*

	$C_j$	0	0	0	0	0	0	1	1	0	0		
$C_i$	$x_i/x_j$	$x_1$	$x_2$	$\lambda_1$	$\lambda_2$	$s_1$	$s_2$	$a_1$	$a_2$	$s'_1$	$s'_2$	bi	$R_i$
1	$a_1$	0.0156	0	-1	0	-1	0	1	0	0	0	1.4052	2,965
1	$a_2$	0	0.474	0	-1	0	-1	0	1	0	0	6.2055	13,092
0	$s'_1$	1	0	0	0	0	0	0	0	1	0	35250	74367.1
0	$s'_2$	0	1	0	0	0	0	0	0	0	1	49838	105143.5
	$Z_j$	0	0	-1	-1	-1	-1	1	1	0	0	8	
	$Z_j - C_j$	0.0156	0.474	-1	-1	-1	-1	0	0	0	0		

All coefficients of each constraint from the *Kuhn Tucker* condition entered into the table. The numbers in row  $Z_j$  are obtained by adding the duplicate results between the coefficients of each side and the number  $C_i$  which is located in the same column. To guarantee the value of  $R_i$ , it is chosen from the line  $Z_j - C_j$ . The variable with the most  $Z_j - C_j$  numbers (minimization problem) is converted into a bi divisor which is then placed into a column  $R_i$ . The numbers  $Z_j - C_j$  in  $x_2$  are the positive characteristics chosen, up to every number in the bi section divided by the number in the variable  $x_2$  to get the number  $R_i$ .

**Table 5.** Determining key rows and columns

	$C_j$	0	0	0	0	0	0	1	1	0	0		
$C_i$	$x_i/x_j$	$x_1$	$x_2$	$\lambda_1$	$\lambda_2$	$s_1$	$s_2$	$a_1$	$a_2$	$s_1'$	$s_2'$	bi	$R_i$
1	$a_1$	0.0156	0	-1	0	-1	0	1	0	0	0	1.4052	2,965
1	$a_2$	0	0.474	0	-1	0	-1	0	1	0	0	6.2055	13,092
0	$s_1'$	1	0	0	0	0	0	0	0	1	0	35250	74367.1
0	$s_2'$	0	1	0	0	0	0	0	0	0	1	49838	105143.5
	$Z_j$	0	0	-1	-1	-1	-1	1	1	0	0	7,611	
	$Z_j - C_j$	0.0156	0.474	-1	-1	-1	-1	0	0	0	0		

In column  $R_i$ , the value in the row  $a_1$  is smaller than the value in the  $R_i$  other columns, so  $a_1$  it is removed and replaced  $x_1$  as a base variable. The rows and columns colored yellow will serve as a reference for the next iteration. Then, the iteration process continues

**Table 6.** First iteration of the *Wolfe model*

	$C_j$	0	0	0	0	0	0	1	1	0	0		
$C_i$	$x_i/x_j$	$x_1$	$x_2$	$\lambda_1$	$\lambda_2$	$s_1$	$s_2$	$a_1$	$a_2$	$s_1'$	$s_2'$	Mrs	$R_i$
0	$x_1$	1	0	-64	0	-64	0	64	0	0	0	90,077	0
1	$a_2$	0	0.474	0	-1	0	-1	0	1	0	0	6.2055	13,092
0	$s_1'$	0	0	64	0	64	0	-64	0	1	0	35160	0
0	$s_2'$	0	1	0	0	0	0	0	0	0	1	49838	49838
	$Z_j$	0	0.474	0	-1	0	-1	0	1	0	0	6.2055	
	$Z_j - C_j$	0	0.474	0	-1	0	-1	-1	0	0	0		

The rows  $x_1$  (green rows) are the rows obtained by dividing the old values with the key numbers ( blue cells ) for calculation  $\frac{a_1}{0,0156}$ . Meanwhile, the value in the row  $a_2$  up to the line  $s_2'$  is obtained by

$Old\ value - (New\ value \times Key\ column)$
--

The old values are the values in the cells in table 4.4, the new values are the values in the rows  $x_1$  in table 6, then the key column is the value in the column  $x_1$  in table 3.4. Please note that calculations are carried out according to the rows and columns.

Values on row  $Z_j$  in table 6 is obtained by adding up the results of multiplying the coefficients of each column with the  $C_i$  values in the same row. Because the value of  $Z_j - C_j > 0$ , the iteration is continued until optimal results are obtained as in the following table 7.

**Table 7.** *Wolfe model* optimal table

	$C_j$	0	0	0	0	0	0	1	1	0	0		
$C_i$	$x_i/x_j$	$x_1$	$x_2$	$\lambda_1$	$\lambda_2$	$s_1$	$s_2$	$a_1$	$a_2$	$s_1'$	$s_2'$	Mrs	$R_i$

0	$x_1$	1	0	-64	0	-64	0	64	0	0	0	90,077	
0	$x_2$	0	1	0	-2	0	-2	0	2	0	0	13,092	
0	$s_1'$	0	0	64	0	64	0	-64	0	1	0	35160	
0	$s_2'$	0	0	0	2	0	2	0	-2	0	1	49825	
	$Z_j$	0	0	0	0	0	0	0	0	0	0	0	
	$Z_j - C_j$	0	0	0	0	0	0	-1	-1	0	0	0	

Based on table 7, the results obtained are  $x_1 = 90,077$ ,  $x_2 = 13,092$ ,  $s_1' = 35160$ , dan  $s_2' = 49825$ : Next, substitute variables  $x_1$  and  $x_2$  into equation 3.2 which is the initial objective function to get the maximum value

$$f(x) = -0,0078x_1^2 - 0,2370x_2^2 + 1,4052x_1 + 6,2055x_2 - 0,0005$$

$$f(x) = -0,0078(90,077)^2 - 0,2370(13,092)^2 + 1,4052(90,077) + 6,2055(13,092) - 0,0005$$

$$= 203.1925$$

**Settlement using the Exterior Penalty Method**

The steps for completing the exterior penalty function method include the following: *First*, form the objective function using the exterior penalty method. Forming an objective function for an unconstrained optimization problem that is in line with the general form of the exterior penalty function method problem, the downsizing problem in equations 3.2 and 3.3, is changed to an unlimited increasing problem by using the exterior penalty function method and creating the objective then z, then choose  $p= 2$  (since 2 is the smallest positive number which means the penalty function always exists at the new value of z after decrementing), the result is.

$$z = f(x) + \mu_k \alpha(x)$$

$$z = f(x) + \mu_k \sum_{i=1}^m [maks\{0, g_i(x)\}]^2$$

Then we obtain the exterior penalty function problem, namely

Minimize

$$z = -0,0078x_1^2 - 0,2370x_2^2 + 1,4052x_1 + 6,2055x_2 - 0,0005 + \mu_k [maks\{0, x_1 - 35250\}]^2 + [maks\{0, x_2 - 49838\}]^2 \tag{3.13}$$

*Second*, determine the solution to the problem of minimizing z, namely  $x^*$

The optimal point will be reached if  $z' = 0$ , then

$$\frac{\partial z}{\partial x_1} = -0,0156 + 1,4052 + 2\mu_k [maks\{0, x_1 - 35250\}] = 0 \tag{3.14a}$$

$$\frac{\partial z}{\partial x_2} = -0,474 + 6,2055 + 2\mu_k [maks\{0, x_2 - 49838\}] = 0 \tag{3.14b}$$

Because the goal of the penalty function problem is to minimize, equation (3.14) becomes

$$\min [-0,0156 + 1,4052 + 2\mu_k(x_1 - 35250)] = 0 \tag{3.15a}$$

$$\min [-0,474 + 6,2055 + 2\mu_k(x_2 - 49838)] = 0 \tag{3.15b}$$

From equation (3.15) obtained:

$$x_1 = \frac{1,4052}{0,0156} = 90,077$$

$$x_2 = \frac{6,2055}{0,474} = 13,092$$

*Third*, investigate whether the value  $x_1, x_2$  is a minimum or maximum value based on sufficient optimality conditions.

If  $\frac{\partial f}{\partial x_j} = 0$ , and  $H(x^*)$  positive definite then  $x^*$  is the minimum point

If  $\frac{\partial f}{\partial x_j} = 0$ , and  $H(x^*)$  negative definite then  $x^*$  is the maximum point

Hessian matrix of function (3.13) are as follows

$$H(x) = \begin{bmatrix} -0,0156 & 0 \\ 0 & -0,474 \end{bmatrix}$$

If  $H(x)$  is expressed in quadratic form, then it is written

$$\begin{aligned} H(x) &= [x_1 \quad x_2] \begin{bmatrix} -0,0156 & 0 \\ 0 & -0,474 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \\ &= -0,0156x_1 \quad -0,474x_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= -0,0156x_1^2 - 0,474x_2^2 < 0 \end{aligned}$$

Based on definition 2.2, matrix  $H(x)$  *definit negatif*, then  $x$  is the maximum point.

Maximum value of

$$f(x) = -0,0078x_1^2 - 0,2370x_2^2 + 1,4052x_1 + 6,2055x_2 - 0,0005$$

For ( $x_1 = 90,077$ ,  $x_2 = 13,092$ )

is

$$\begin{aligned} f(x) &= -0,0078(90,077) - 0,2370(13,092) + 1,4052(90,077) \\ &\quad + 6,2055(13,092) - 0,0005 \\ &= 203.1925 \end{aligned}$$

### Comparison with previous research

One of the foundations used in conducting research on Rice Crop Optimization in Simalungun Regency using Quadratic Programming and the Exterior Penalty Function Method is research that has been carried out previously. In this research, the topic used is productivity optimization in the agricultural sector regarding food crops, in this case rice plants using the quadratic programming method and the penalty function method. So the reference used is research that is still related to that topic.

Quadratic research was carried out by Insani (2017) who expanded food crops in the city of Magelang by involving quadratic programming and the exterior penalty method which obtained estimated results using simplex calculations with the majority of the production of all food crops from rice, cassava and corn, namely 387,0586 tons with a rice harvest around 520.75 hectares, cassava harvest covering 33,6426 hectares, and large corn harvest 8,4817 hectares r. Meanwhile, if calculated using the exterior penalty method, the average rice harvest is 520.75 hectares, cassava harvest is 33.6426 hectares, and corn harvest is 8.4817 hectares, so the total food crop reaches 387.0586 quintals.

Further research was conducted by Larita, et al. (2018) who improved the total average rice production in West Kalimantan using the *Wolfe strategy* for a quadratic program which produced 323.82 kw/Ha.

Next, Safani and Sari (2020), must carry out research to increase rice and corn production in the South Coast region by using an exterior penalty function strategy which results in total rice and corn production being 412.65 Kw or Ha with a plant area of 83,333 Ha and a big corn which covers an area of 16,667 Ha.

RH Putri, et al (2021) directed research regarding testing the guarantee of the largest portfolio proportion in a quadratic program using the Wolfe technique which utilizes the most extreme stock portfolio structure, especially the Frederick S. type by utilizing 2 stock models, namely Bank Skillet Indonesia and Bank Focal Asia Tbk, then the largest proportion figure was Bank Dish Indonesia (PNBN) at 23.57% while Bank Focal Asia Tbk (BBCA) was 76.4%.

Then, the next research was carried out by Hikmah (2022) who expanded cassava and soybean production in West Pasaman Regency by utilizing *Wolfe's* quadratic programming technique. The increase in yield with cassava & soybean production in West Pasaman Regency using the Wolfe quadratic program strategy was 406.31 Kw or Ha, with the highest cassava harvest area of 208.50 Ha & soybeans 230,875 Ha.

**Table 8.** Research Comparison Table

Author	Year	Language and Objects	Method used	Research result
Human	2017	Optimization of food crops: rice, cassava and corn	Quadratic programming and the exterior penalty function method	The results are the same for both methods with an average total production of 387.0586 quintals. The rice harvest area is 520.75 hectares, the cassava harvest area is 33.6436 hectares and the corn harvest area is 8.4817 hectares
Anni Larita, Helmi and Yudhi	2018	Optimizing rice production. The object is rice harvesting	Wolfe method quadratic programming	The optimal average yield of rice production obtained is 323.8276658 kw/Ha
Emmelia Safani and Devni Prima Sari	2020	Optimizing the production of rice and corn food crops	Exterior penalty function method	The optimal average production results for the rice plant area are 66667 Ha, the corn plant area is 16667 Ha and the average total production for both is 236.56 kw/Ha
RH Putri, Sudarwanto, and ED Wiraningsih	2021	Analysis of determining optimal portfolio proportions with Bank Pan Indonesia (PNBN) and Bank Central Asia Tbk (BBCA) objects	Quadratic programming with the <i>Wolfe model</i>	The results obtained are in decimal form which is a percentage of shares of 23.57%, for PNBN it is 76.4% while for BBCA the expected return value of one year's investment funds is 52%
Nurul Hikmah and Defri Ahmad	2022	Optimization of average cassava and soybean production	<i>Wolfe</i> method quadratic programming	The average optimal production of cassava and soybeans is 340.95 kw/Ha. The cassava harvest area is 255.38 hectares and

Author	Year	Language and Objects	Method used	Research result
				soybeans is 469.5 hectares

## CONCLUSION

Mathematical model for optimizing average rice production in Simalungun Regency, namely maximizing the objective function:

$$f(x) = -0,0078x_1^2 - 0,2370x_2^2 + 1,4052x_1 + 6,2055x_2 - 0,0005$$

with obstacles

$$g_1(x) = x_1 \leq 35250$$

$$g_2(x) = x_2 \leq 49838$$

$$x_1, x_2 \geq 0$$

*Wolfe* model simple quadratic programming calculations and the exterior penalty function method, the results obtained were the same as the average total production of lowland and field rice crops of 203.1925 tons with a  $x_1$  90,077 hectare of wetland rice harvest area, and a field harvest area of 13.092 hectare.

## RECOMMENDATION

In this proposition, we have evaluated how to create a nonlinear form for overall rice production in Simalungun Regency and its arrangement involving quadratic programming as well as the exterior penalty function method. The final treatment of quadratic programming uses *Wolfe structures*. Based on graph 4.1, overall production has not been maximized from 2016 to 2022. Not only that, the harvest area experiences fluctuations every year due to shrinkage and instability. Therefore, the government and society are encouraged to work together to increase rice production in Simalungun Regency.

Farmers can increase productivity by managing agricultural land well so that agricultural yields can increase. Then, farmers can also learn about planting using various methods that can increase the amount of production with minimum costs, such as using surrounding materials, for example making their own compost.

The government must also participate in helping increase the productivity of rice plants by improving the welfare of farmers. This effort is carried out by ensuring the distribution of fertilizer and seeds at stable prices, holding agricultural extension services to support farmers to better understand agricultural science.

For readers who are interested in conducting research using nonlinear model optimization methods, there are several methods, including: the *Karush Kuhn Tucker method*, the *Separable method*, the exterior penalty function method and so on. There are also modern methods such as *Ant Colony Optimization (ACO)*. Solving using quadratic programming can also be solved using the *Primal-Dual Interior Point algorithm*, apart from that, the formation of the objective function, other than using the least squares method, can use the *Singular Value Decomposition (SVD)* method.

## REFERENCES

- A. M. Geoffrion. (1971). Duality in Nonlinear Programming. SIAM Review. 23-26
- BPS, B. P. (2021). *Kabupaten Simalungun dalam angka*. Kabupaten Simalungun: BPS Kabupaten Simalungun.
- Budiwati, G. A., Kriswiyanti, E., & Astarini, I. A. (2019). Aspek Biologi Dan Hubungan Kekerabatan Padi Lokal (*Oryza sativa* L.) Di Desa Wongaya Gede Kecamatan Penebel, Kabupaten Tabanan, Bali. *Metamorfosa: Journal of Biological Sciences*, 277.

- Erlina, Syaripuddin, & Tisna Amijaya, F. D. (2022). Penyelesaian masalah pemrograman kuadratik menggunakan metode beale (studi kasus : data produksi kota balikpapan). *jurnal EKSPONENSIAL*, 87.
- Indainanto, Y. I., Nasution, F. A., Fauzan, I., Ardian, M., & Dalimunthe, M. A. (2022). Analisis Jaringan Interaksi Momentum Hari Tani Di Media Sosial. *Jurnal Kajian Agraria dan Kedaulatan Pangan*, 8.
- Insani, S. N., & Sari, E. R. (2017). Optimasi tanaman pangan di kota magelang dengan pemrograman kuadratik dan metode fungsi penalti eksterior. *Jurnal matematika*, 41-42.
- K. G. Murty, dan F. T. Yu (1998). *Linear Complimentarity, Linear and Nonlinear Programming*. University of Michigan. 24-26
- Larita, A., Helmi, & Yudhi. (2018). Optimasi Rata-Rata Produksi Padi Kalimantan Barat. *Buletin Ilmiah Math. Stat. dan Terapannya (Bimaster)*, 200.
- M, M., & Subanar. (2017). Kajian terhadap Beberapa Metode Optimasi (Survey Of Optimization Method. *JUITA*, 47-50.
- Putri, R. H., Sudarwanto, & Wiraningsih, E. D. (2022). Analisis penentuan portofolio optimal pada quadratic programming dengan menggunakan metode wolfe. *PRISMA, PROSIDING SEMINAR NASIONAL MATEMATIKA*.
- Ruvananda, A. R., & Taufik, M. (2022). Analisis faktor-faktor yang mempengaruhi impor beras di Indonesia. *KINERJA: Jurnal Ekonomi dan Manajemen*, 196.
- Safani, E., & Sari, D. P. (2020). Optimasi Produksi Tanaman Padi dan Jagung di Kabupaten Pesisir Selatan Menggunakan Metode Fungsi Penalti Eksteior. *UNPjoMath*, 45.
- Sitanggang, R. P. (2023). Analisis Optimasi Program Kuadratik Dengan Fungsi Penalty. *Jurnal Riset Rumpun Ilmu Pendidikan*, 34-35.
- Utama, Z. H. (2015). *Budidaya Padi pada Lahan Marjinal*. Yogyakarta: ANDI.
- W. Hua, "APPLICATION OF THE REVISED SIMPLEX," 1998, Hangzhou University, 1990:4-7
- Wright, Stephen J. (2015), "Optimasi Berkelanjutan (Linear and Nonlinear Programming)", dalam Nicholas J. Higham;dkk. *Pendamping Princeton untuk Matematika Terapan*, Princeton University Press, 281-293